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TRANSIENT THERMAL STRESSES WITH TEMPERATURE AND PHASE DEPENDENT PROPERTIES

A THESIS

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This thesis is affectionately dedicated to my parents, Mr. and Mrs. Richard D. Herron, for without their help and encouragement this work would have been impossible.

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SUMMARY

Presented in this thesis are the principles necessary to the calculation of a temperature gradient and thermal stresses by numerical methods. In particular a program was written for use with Burroughs 5500 Computer that will calculate thermal stresses for the general case of a free flat steel plate as it is cooled (or heated) from some uniform initial temperature. The program was written so that the stresses could be calculated with constant physical properties or with physical properties as a function of temperature and phase.

Using this program stresses were calculated for six different cases. The results of the calculations showed that stresses calculated using phase and variable properties differ from those calculated with just variable properties at the beginning of cooling, but that the residual stresses were approximately the same. Also the calculations made with variable properties resulted in a decrease in size of the material, while the calculations made with constant properties resulted in an increase in size. Another result of the calculations was that if the material was considered to be only elastic the resulting stresses were several times larger than those made with either variable or constant properties having plastic flow.

CHAPTER I

INTRODUCTION

Methods for the calculation of thermal stresses have been well formulated (1) since the existence of such stresses presents a major problem in structures subject to high temperatures. Thermal stresses arise if there is a difference in the expansion of connected components of the structure such as might be caused by the components being at different temperatures or being at the same temperature but having different coefficients of expansion. Stresses may also exist in a single component if there is a temperature gradient or a nonhomogeneous microstructure.

For the simplest case of thermal stress calculation, the properties are considered to be constant. The assumption of constant properties is erroneous, however, since the properties of a metal change with temperature. For example, in steel, the coefficient of thermal expansion and specific heat increase with increasing temperature while the yield strength and modulus of elasticity decrease. Also, under the condition of changing temperature, a metal may undergo a phase transformation; not only do different phases have different properties, but there is also an expansion and liberation of latent heat associated with the transformation. The above discussion of the effect of variable properties has been mentioned with respect to the calculation of stresses, but before the stresses can be determined the temperature distribution must be

known, obviously the effect of temperature on properties must also be considered in these calculations.

Another factor that must be considered in the calculation of thermal stresses is the effect of plastic flow. While the theory for thermal stresses with elastic-plastic deformation has been developed and explored (2), for the practicing engineer this theory is quite complex and time consuming. This presents the need for simplified methods that can be easily programmed for use on a digital computer.

From the above discussion it can be seen that the problem of calculation thermal stresses really involves two problems: (1) the calculation of temperature distributions, and (2) the calculation of thermal stresses from this temperature gradient. It can also be seen that these calculations can be performed using either constant or variable properties depending upon the accuracy desired.

CHAPTER II

FORMULATION OF THE PROBLEM

Defining the Problem

The objective of this work is to investigate the effect of using variable physical properties in place of constant properties in the calculation of thermal stresses and to approximate the effect of cooling rates and phase change on these calculations. As part of this investigation a computer program was written, for use with the Burroughs 5500 computer, that will calculate the stresses produced by rapid cooling. The program was written in such a general form that either constant or variable properties can be used.

The case considered was the general one of a free flat plate. A free flat plate is one that is supported in such a manner that there will not be any external forces or moments acting upon it. The flat plate was chosen for study because with it the principles involved can easily be illustrated and yet it is general enough that the principles can be applied to other more complicated configurations. As in most analysis the material will be assumed to be homogeneous at the beginning of the analysis, also to simplify the discussion the material will be assumed to be isotropic. This means that the elastic properties are the same in all directions. Although the assumption of an isotropic material may not be completely correct for a flat plate, since a process

such as rolling may cause the crystals of the metal to be orientated in a certain manner, it will be a valid assumption for most cases.

The plate will be limited to the particular case shown in Figure 1, where L, the length, is many times greater than W, the width.

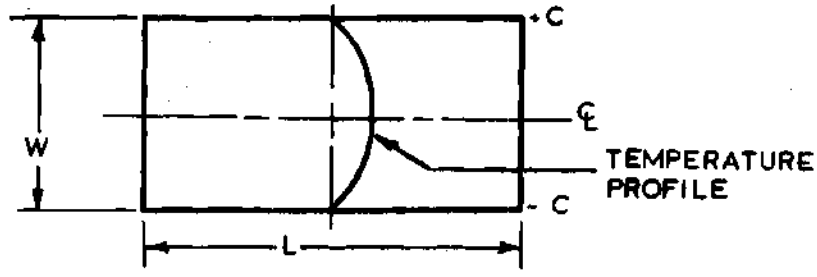

Figure 1. Plate Configuration

The thickness of the plate is very small compared to either the length or the width. Starting with the entire plate at some uniform initial temperature, heat will be removed through the edges along the length of the plate. There will be no heat loss through the width edges and the sides of the plates as these areas will be assumed to be perfectly insulated. This configuration will result in a temperature gradient across the width of the plate and uniform temperatures along the length and through the thickness. By making these limitations upon the plate configuration and temperature distribution, the problem is reduced to one-dimension stress, that is, the stresses in the width direction and thickness direction will be zero, thus the only stress will be in the direction along the length.

The solution can easily be modified to a two-dimensional stress case by using equations that can be found in books by Timoshenko (3) and Manson (2). For the two-dimensional problem the plate would be cooled from the faces thus producing a temperature gradient through the thickness. The stresses produced would then be along the width and length of the plate. Since the two-dimensional solution is a modified one-dimensional solution and the added equations needed for the two-dimensional stress do not change the method or have any effect upon the comparison of results with fixed and variable properties, the one-dimensional case will be adequate for the study.

Previous Works

Now that the problem has been defined from the physical standpoint, let us investigate some of the work of other authors. This survey will be done in two parts, one dealing with heat transfer and the second with thermal stresses. Neither of these areas will be explored in this chapter with an emphasis on theory or equations, for this study will be handled in later chapters.

Heat Transfer

The problem of heat transfer calculations by numerical methods is very common and almost every book on heat transfer will explain a numerical method. However, some very complete information can be found in works by George M. Dusinberre. In an article entitled "Numerical Methods for Transient Heat Flow" (4) he extends the generality of the application of numerical methods in three ways: (1) A modulus is developed, the choice of which will determine if the calculations

proceed rapidly or if they proceed slowly with greater precision; (2) a criteria is developed to insure that the choice of the modulus will produce convergence. That is most important when there is surface convection; (3) a method for handling conductivity and specific heat when they vary independently with temperature is developed. This article also includes two examples to demonstrate the three extensions, one dealing with a slab of cast iron that is being cooled, the other with a cylindrical surface that is being heated.

A more complete discussion by Dusinberre can be found in a book titled *Heat-Transfer Calculations by Finite Differences* (5). In this book he goes into more detail for the calculations of transient and steady state temperature gradients for one or multidimensional, simple or complex systems. Also covered are some methods for flow and variable properties, thus the book is a fairly complete reference for numerical methods of solving heat transfer problems.

A difficult problem encountered in any heat transfer problem is that of determining value of film or surface conductance. In an article by V. Paschkis and G. Stolz, Jr. (6) an attempt was made to evaluate this coefficient, but because of the complexity of the problem very little useful data was obtained.

Thermal Stress

A paper written by John L. Maulbetsch (7) presents a solution to a two-dimensional thermal stress problem. The solution is for a plate of uniform thickness that has its two sides maintained at different uniform temperatures. The plate is simply supported at the edges and

is of a homogeneous and isotropic material, thus, although Maulbetsch considers all of his properties constant, his solution is a good basis for extending the one-dimensional solution presented here to two dimensions.

In 1954 H. H. Hilton presented a paper (8) that offered an analysis of thermal stresses and strains due to a steady-state temperature gradient in a thickwalled cylinder and a thin circular plate, taking into account the variations of the elastic shear modulus and the coefficient of thermal expansion with temperature. His results showed that the maximum thermal stresses were lower and that the maximum thermal strains were higher for the calculations made with temperature-dependent properties than with constant properties. Hilton did not try to account for either phase change or plastic deformation, but limited himself to elastic deformation with variable properties.

A later article written by J. S. Born and G. Horvay (9) explained the calculation of thermal stresses in a rectangular strip for a temperature gradient along the strip, but they did not try to make any allowances for temperature dependent properties.

One of the best methods for calculating thermal stresses with plastic deformation was developed by A. Mendelson and S. S. Manson for the National Advisory Committee for Aeronautics (10). In the paper they presented a method for solving plastic deformation problems in the elastic-plastic range. The method is one of successive integrations in which the integral describing the situation is evaluated as if there were no plastic strain, a plastic strain is then found and this value is

then used in the integral, the process is repeated until successive answers agree to the desired accuracy. Four examples are given to demonstrate this method, the first three are for small plastic strains and the fourth is with plastic strains of about 1 per cent. Because of the simplicity of the method it lends itself well to making calculations using variable properties.

Another useful method is one that was developed by D. M. Gilbey (11) for the calculation of thermal stresses in a graphite plate. Using Gilbey's method one thinks of the plate as being made up of many thin laminations; starting from the initial unstressed state the stress in the final state is calculated by finding the stresses in several completely hypothetical states, somewhat in the way one calculates the heat that is absorbed in a complicated process from the thermodynamic data. While this method was developed for graphite it can easily be used for metals, although it is not as useful as the method formulated by Mendelson and Manson.

Summary

As can be seen there has been much work done on calculating thermal stresses with some people taking into consideration temperature-dependent properties and also stresses in the elastic-plastic region. Thus far no one has tried to in any way consider the effect of phase change on thermal stresses and in particular phase change with respect to cooling rates. This, then, brings up two questions:

1. How can the effect of rapid cooling and phase change be accounted for?

2. How do the stresses calculated considering phase change and variable properties compare with stresses calculated using only variable properties, and how do they compare with those made using constant properties?

CHAPTER III

METHOD OF SOLUTION FOR HEAT TRANSFER PROBLEMS

Basic Laws

For the calculation of the transient temperature gradient, we will make use of the first two laws of thermodynamics. For the purpose of heat transfer these two laws can be stated as:

1. The energy supplied to a system is equal to the energy stored in the system plus the energy that is removed from the system. The energy that is stored in the system may be either positive or negative and the equation for the system may be stated in terms of time.

2. It is only possible for energy to be transferred by virtue of a negative temperature gradient, that is, from a high temperature particle to a low-temperature particle.

Heat Transfer Definitions

In order to formulate equations for heat transfer, certain quantities must be defined. The first of these will be heat capacity. If we let Q be heat energy in units of BTU and t be temperature in degrees Fahrenheit, then we can say that the heat capacity, C, of the system is

$$C_{12} = \frac{Q_2 - Q_1}{t_2 - t_1} = \frac{Q}{\Delta t} \quad \frac{\text{Btu}}{^{\circ}\text{F}} \tag{3.1}$$

where Δt is the temperature increase of the system.

Now if we let the system be of a homogeneous material and let w be the weight of this system, then we can define specific heat, c, for the system as

$$u = \frac{Q}{w} \qquad \text{Btu/lb} \tag{3.2}$$

$$c = \frac{u}{\Delta t} \qquad \text{Btu/lb } ^\circ\text{F} \tag{3.3}$$

where Δt denotes a temperature increase that results from heat transfer.

Thus far in this discussion we have made no mention of cooling rate, but in order to study transient effects, rate must be considered. If we now let q be energy per unit time or Btu per hour, we can define conductance, K, of a system as

$$K_{12} = \frac{q}{t_1 - t_2} \qquad \text{Btu/hr } ^\circ\text{F} \tag{3.4}$$

where $t_1 - t_2$ is the temperature difference in the system causing the heat transfer.

If we use the symbol θ to denote the temperature difference $(t_1 - t_2)$ and give it a negative sign, since the heat transfer is opposite to the temperature distribution, we then have

$$K_{12} = \frac{q}{\theta} \qquad \text{Btu/hr } ^\circ\text{F} \tag{3.5}$$

If we call the length of the path of heat transfer, ℓ, and the cross-sectional area through which it flows, A, then we can define what is known as thermal conductivity, k, as

$$k_{12} = \frac{q}{A(-\theta/\ell)} \quad \text{Btu/(hr)(sq ft)(°F/ft)} \tag{3.6}$$

Both the properties, specific heat and thermal conductivity, can be written in the form of a general function of temperature as

$$c = du/dt$$

or

$$u = \int c\, dt \tag{3.7}$$

and

$$k = \frac{q}{A(-dt/d\ell)} \tag{3.8}$$

The concepts of thermal conductivity above were made for solids or materials in which the molecules remain in a relative fixed position.

Next consider heat transfer between a solid substance and a fluid medium. For this situation the quantity of heat transferred is dependent upon the properties of the fluid by means of a factor called the surface or film coefficient.

As with thermal conductivity, we will have q Btu per hour that is transferred through some area A in square feet across some temperature difference $t_1 - t_2$ as a conductance

$$H_{12} = \frac{q}{t_1 - t_2} = \frac{q}{-\theta} \qquad \frac{\text{Btu}}{\text{hr } ^\circ\text{F}} \tag{3.9}$$

or we can express it as a film coefficient as

$$h_{12} = \frac{q}{A(-\theta)} \qquad \frac{\text{Btu}}{(\text{hr})(\text{Sq ft})(^\circ\text{F})} \tag{3.10}$$

It can be seen that the value of h will depend upon the temperature of the fluid, the thermal properties of the fluid and the geometry of the system.

Thus far we have cited the transmission of energy by conductance (conductivity) and by convection (surface or film coefficient), but energy can also be transmitted by radiation. Since this last mode of heat transfer is not directly related to the problem being studied, it will only be mentioned and not discussed.

Numerical Method for Heat Transfer Calculations

Now using these modes of heat transfer, we can develop general finite-difference equations by which the temperature and its gradients can be calculated for any point in the system. The normal approach is to calculate the temperature by an appropriate integration, but when using a numerical solution only a select number of temperatures are calculated at certain points. In this way each point is used to represent a region of the system that includes that point. For this problem it will be assumed that the system is homogeneous, and of regular form so that each region can be given definite geometrical shape, such as a

square or cube. This makes the numerical method a form of trapezoidal integration of the analytical solution.

To begin the analysis, the energy input into a region will be considered. Let region *M* be the region under consideration, if region *M* is receiving heat from another region *N* by conduction then the rate of heat transfer can be expressed as

$$q_{NM} = K_{NM}(t_N - t_M) \qquad \text{Btu/hr} \tag{3.11}$$

where for a homogeneous material of rectilinear geometry

$$K_{NM} = \frac{kA}{\ell} \tag{3.12}$$

Next, if region *M* receives heat from region *L* by convection, the heat transfer rate can be stated as

$$q_{LM} = K_{LM}(t_L - t_M) \qquad \text{Btu/hr} \tag{3.13}$$

where

$$K_{LM} = hA \tag{3.14}$$

We will call any heat received by region *M* by radiation q_{DM} and any heat generated within the region q_{GM}. The total heat exchanged with region *M* can be expressed as the sum of the heat transferred by each of the modes; that is

$$q_M = q_{NM} + q_{LM} + q_{DM} + q_{GM} \qquad \text{Btu/hr} \qquad (3.15)$$

The energy stored in the region can be calculated by making use of the specific heat of the region. Thus, for some time interval $\Delta\tau$, where the temperature of the region at the beginning of the time interval is t and the temperature at the end is t', the heat transfer rate can be stated as

$$q_M = \frac{c_M(t'_M - t_M)}{\Delta\tau} \qquad \text{Btu/hr} \qquad (3.16)$$

where c_M is the specific heat for region M. It is possible now to say that the two q_M are equal, since the first gives the heat rate at a given instant in time and the second gives the heat rate over a certain period of time. There are conditions under which equating the two q_M will give very erroneous results, but if the region and time divisions are allowed to pass to a limit the result will be quite satisfactory (5). Equating the two equations we have

$$q_{NM} + q_{LM} + q_{DM} + q_{GM} = \frac{c_M(t'_M - t_M)}{\Delta\tau} \qquad \text{Btu/hr} \qquad (3.17)$$

For the problem under consideration in this thesis q_{DM} and q_{GM} are equal to zero, so expanding the above equation

$$q_{NM} + q_{LM} = \frac{C_M(t'_M - t_M)}{\Delta\tau}$$

$$K_{NM}(t_N-t_M) + K_{LM}(t_L-t_M) = \frac{C_M(t'_M-t_M)}{\Delta\tau}$$

$$\frac{K_{NM}\Delta\tau}{C_M} t_N + \frac{K_{LM}\Delta\tau}{C_M} t_L - \frac{(K_{NM} + K_{LM})\Delta\tau}{C_M} t_M = t'_M - t_M$$

or

$$t'_M = \frac{K_{NM}\Delta\tau}{C_M} t_N + \frac{K_{LM}\Delta\tau}{C_M} t_L + \left[1 - \frac{(K_{NM} + K_{LM})\Delta\tau}{C_M}\right] t_M \qquad (3.18)$$

An equation like the one just derived can be written for each region of the system. Thus, given the initial temperatures and the boundary conditions of the system, the temperature distribution can be calculated over any number of $\Delta\tau$'s.

In Equation (3.18) all of the components will be positive with the possible exception of

$$\left[1 - \frac{(K_{NM} + K_{LM})\Delta\tau}{C_M}\right] t_M \qquad (3.19)$$

This component will be negative if the choice of $\Delta\tau$ is large, but if this was allowed to occur the results would be ridiculous. If this quantity was negative it would imply that the hotter the region M is at the start of the time interval, the colder it will be at the end of the interval. This is not possible in the real case so as a condition for $\Delta\tau$ we have

$$\frac{(K_{NM} + K_{LM})\Delta\tau}{C_M} \leq 1$$

or

$$\Delta\tau \leq \frac{C_M}{K_{NM} + K_{LM}} \tag{3.20}$$

Since $\Delta\tau$ is to be the same for all regions of the system, this criterion must be satisfied for each region. If we now expand this relationship, that is, let

$$K_{NM} = \frac{k_{NM}A}{\Delta x} \tag{3.21}$$

$$K_{LM} = \frac{K_{LM}A}{\Delta x} \tag{3.22}$$

$$C_M = A\ \Delta x\ c\rho \tag{3.23}$$

inserting these into the stability criterion equation

$$\Delta\tau \leq \frac{A\ \Delta x\ c\rho}{\frac{k_{NM}A}{\Delta x} + \frac{k_{LM}A}{\Delta x}} = \frac{A\ \Delta x\ c\rho}{\frac{A}{\Delta x}(k_{NM} + k_{LM})}$$

$$\Delta\tau \leq \frac{(\Delta x)^2 c\rho}{k_{NM} + k_{LM}} \leq \frac{(\Delta x)^2 c\rho}{2k} \tag{3.24}$$

If we let the specific heat C be its smallest value in the system and thermal conductivity K have its largest value, then any $\Delta\tau$ that is equal to or less than the one found from the above equation will work for the whole system.

Property Variation with Temperature

At this point, let us consider the effect of temperature on the properties and behavior of the material. Low carbon steel, such as AISI C1020, will be used in this discussion, but the method presented is general enough that it can easily be applied to other materials.

The usual kind of phase diagram does not give any indication of the time dependency of the phase change. But if samples of a given steel are quenched from some initial high temperature to some temperature below the eutectoid temperature and held at that temperature for a set period of time and then quenched to room temperature more information can be found (12). The extent of transformation may then be determined by studying metallographic sections of the samples. From this data, a time-temperature-transformation curve, or T-T-T curve, can be drawn.

A T-T-T curve, also sometimes called an S or C curve because of its shape, is shown in Figure 2 (13). The curves are labeled to denote the start and finish of the austenite decomposition. Depending upon the rate of cooling the phase transformation of the metal will occur at different temperatures. For example, if the metal is cooled very rapidly, as along curve C1, the phase transformation will not occur until the formation of Martensite which occurs at about 433°F; if the metal is cooled very slowly as along curve C3, the phase transformation

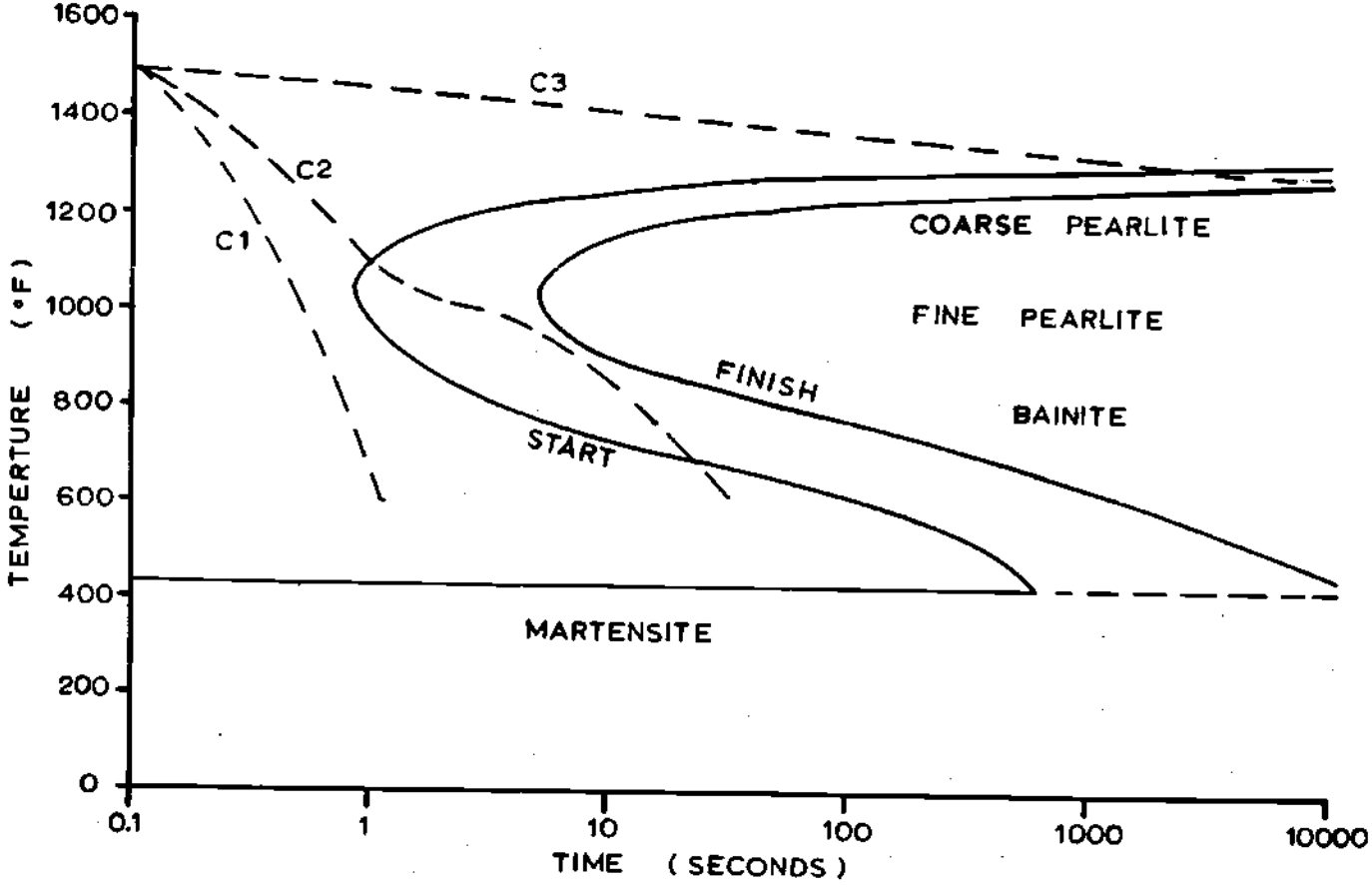

Figure 2. T-T-T Curve for Low Carbon Steel

will occur at about 1330°F, and finally if the rate of cooling is along curve C2 the temperature of the phase change will be approximately the intersection of C2 and the T-T-T curves.

When a metal is slowly heated, the T-T-T curve does not exist so the phase transformation occurs at the same temperature that it would occur if the metal was cooled along line C3, that is 1330°F. The property of the metal that is most affected by a phase change is the heat capacity of the metal. As the metal is heated, the metal reaches a temperature at which the phase change begins and as more heat is added there is little or no change in the temperature. If still more heat is added, the temperature will once again begin to rise. The heat that is added to the metal, but which result in little or no change in temperature, is called latent heat. This phenomenon can be seen in the specific enthalpy curve, Figure 3 (14).

Now that some basic information about cooling rates and heat capacity has been explained, let us return to the problems of calculating the temperature gradient, but this time we will consider the properties of the material to vary with temperature.

In the previous equation relating the energy stored to the energy received by a system there were three quantities that could vary with temperature, thermal conductivity, film coefficient, and specific heat.

Let us start with film coefficient. A forced film coefficient normally does not vary much with temperature, although it will be strongly influenced by the flow rate, but conditions can be imposed so that it will be constant (5). Thus, it is possible to consider the film

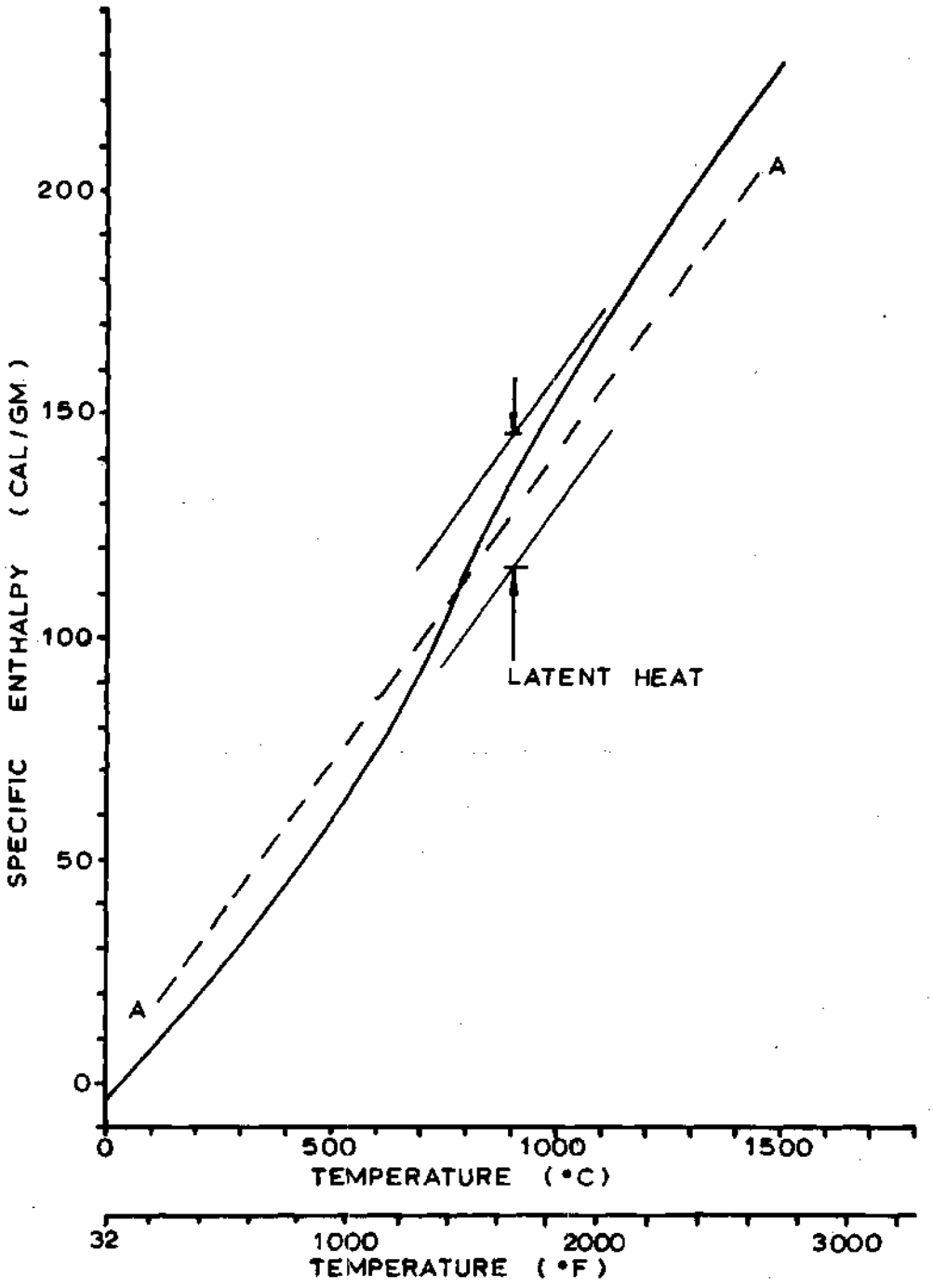

Figure 3. Specific Enthalpy Curve for AISI C1020 Steel

coefficient to be a constant.

The thermal conductivity of the material can be very dependent upon temperature and as a result if the temperature gradient is large then considerable error could result from the use of a constant coefficient of thermal conductivity. There is a change in slope of the conductivity versus temperature curve, but this change occurs well above the temperature at which a phase change takes place, therefore it can be concluded that this change in slope is not the result of a phase transformation. Thus, the rate of cooling will have little or no effect upon the thermal conductivity curve. This means that as the metal is cooled, and as each point of the metal assumes a different temperature, the same conductivity curve will still apply to each point. Therefore, in order to incorporate thermal conductivity as a variable into the calculation of a temperature gradient by a numerical method it is only necessary to average the temperature of the region being studied with the temperature of the adjacent region to which energy is being transferred. With this average temperature, a value of thermal conductivity can be found and this value used for that specific calculation. This procedure is then repeated for each calculation made.

Like thermal conductivity, the specific heat of a metal is dependent upon temperature. Also, if the concept of total enthalpy is considered, it will be found that the enthalpy is not only related to temperature but also to phase changes. If we let i be the symbol for specific enthalpy (Btu/lb) and I be total enthalpy we can write:

$$q_M = \frac{I'_M - I_M}{\Delta\tau} \quad \frac{Btu}{hr} \tag{3.25}$$

Now using this concept, specific heat can be defined as:

$$c = \frac{di}{dt} \quad \frac{Btu}{lb°F} \tag{3.26}$$

$$C = \frac{dI}{dt} \quad \frac{Btu}{°F} \tag{3.27}$$

or specific enthalpy

$$i = \int c\, dt \quad \frac{Btu}{lb} \tag{3.28}$$

Now because of the phenomenon of latent heat, a condition where enthalpy is increased (heat absorbed) without a change in temperature, the above equation is incomplete. To be complete it should be

$$i = \int c_1 dt + L_2 + \int c_3 dt + Lq + \cdots \tag{3.29}$$

where the c's are the specific heats and the L's are latent heats.

Figure 3 is a curve of this form with the latent heat effect showing up at about 1330°F for the phase change in steel. The shape of the curve is dependent upon the temperature at which the phase change occurs and this temperature is dependent upon the rate of cooling. Normally in a cooling problem each point across the material will have a different cooling rate. This means that each point will also have a

different curve relating specific enthalpy to temperature. The question now arises as to what will be the shape of the specific enthalpy curve for each of the different cooling rates. A good approximation of the specific enthalpy curve for different cooling rates can be had by moving the normal curve along a line of inversion (15), line A-A in Figure 3. In the normal curve the latent heat resulting from a phase change occurs for steel at about 1330°F, but as mentioned before with different cooling rates the phase change occurs at lower temperatures, so if the curve is moved so that the latent heat is now at the temperature of the phase changes a good estimate of the actual curve should result.

Temperature Calculations with Variable Properties

The energy transferred, q_a in Btu/hr, was earlier written as

$$q_M = \frac{C_M(t'_M - t_M)}{\Delta\tau} \quad \frac{Btu}{hr} \tag{3.30}$$

and then using this equation an energy balance was made. Now we know that the energy transferred can also be stated as

$$q_M = \frac{I'_M - I_M}{\Delta\tau} = \frac{\Delta I_M}{\Delta\tau} \quad \frac{Btu}{hr} \tag{3.31}$$

If we make the same energy balance as before, using the same reasoning, we can write

$$K_{NM}(t_N - t_M) + K_{LM}(t_L - t_M) = \frac{\Delta I_M}{\Delta\tau} \tag{3.32}$$

Using this equation the temperature of any region over a period of time can be calculated. The first step is to calculate the total enthalpy of the region. This is done by finding the specific enthalpy of that region at its initial temperature, this can be found from the specific enthalpy-temperature curve for that region and then multiplying this by the mass of that region. The second step is to calculate the change in enthalpy by the equation

$$\Delta I_M = \Delta\tau[K_{NM}(t_N - t_M) + K_{LM}(t_L - t_M)] \qquad (3.33)$$

where $\Delta\tau$ must satisfy the stability criterion mentioned before. The last step is to add the total enthalpy to the change in enthalpy and divide by the mass; then using this value work backwards on the specific enthalpy-temperature curve to find the new temperature of the region.

This procedure is repeated for each region of the system to find the new temperature of the system after $\Delta\tau$ time. This can then be repeated many times to find the temperature of the system after any number of $\Delta\tau$'s.

CHAPTER IV

METHOD OF SOLUTION FOR THERMAL STRESS PROBLEM

Basic Relationships

A major cause of thermal stresses in any structure is non-uniform temperature distribution. With any rise in temperature the body will expand, but if the rise is not uniform throughout the body each portion will expand a different amount. Since each portion is interconnected, its free expansion will be retarded by adjacent portions, thus producing a stress in that portion.

Many thermal stresses problem can be reduced to a simple problem of boundary conditions. The case of a free thin flat plate with uniaxial stress is of this type. If a thin flat plate is given a temperature distribution across its width and has uniform temperature along its length and through its thickness and is simply supported, it is essentially a uniaxial stress case. It is assumed that this case can be approximated by considering the plate to be constructed of a system of bars which are rigidly connected together at the ends. This analogy is commonly used in the consideration of an all elastic problem and has been used by A. Mendelson and S. S. Manson for the plastic case (10). While this analogy is not exact it is hoped that it does provide a good solution for stresses in an area that is not near the ends of the plate. Thinking of the plate in this way, the plate is considered to consist of a number of elements (bars) as shown in Figure 4 on the following page.

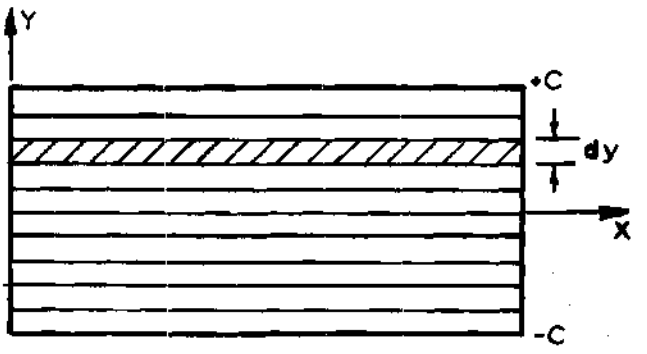

Figure 4. Analogous Plate

Thermal expansion of any element of the plate can be expressed as

$$\frac{\Delta L}{L} = \alpha T \tag{4.1}$$

where ΔL is the change in length of the element, α is the coefficient of thermal expansion and T is the change in temperature.

It can be seen that if the element is fixed so that expansion is impossible, the stress in the element would be

$$\sigma = -\alpha T E \tag{4.2}$$

But, since the plate is free to expand, there will be no stresses as long as the temperature change T across the plate is a constant.

The force across the end of the plate can be expressed by the integral

$$F = \int_{-C}^{C} \sigma \, dy \tag{4.3}$$

but since the plate is free to expand, this force is zero. Also, the moment on the end of the plate can be expressed as

$$M = \int_{-C}^{C} \sigma y \, dy \tag{4.4}$$

but like the force, the moment is also zero. Thus, these two equations form the boundary conditions

$$\int_{-C}^{C} \sigma \, dy = 0 \tag{4.5}$$

and

$$\int_{-C}^{C} \sigma y \, dy = 0 \tag{4.6}$$

In the bar analogy planes that are perpendicular to the x axis remain planes after heating because the ends are rigidly held in a plane. For the analogous case of a plate it will be assumed also that plane sections will remain plane and thus strain can be expressed as

$$\varepsilon_X = a + by \tag{4.7}$$

where a and b are constants. Another relationship for strain is that the total strain can be expressed as the sum or stress, thermal expansion, and plastic strain.

$$\varepsilon_X = \frac{1}{E}\sigma_X + \alpha T + \varepsilon_{XP} \tag{4.8}$$

If this equation is rearranged so that stress is expressed in terms of the other quantities, we have

$$\sigma_X = E(\varepsilon_X - \alpha T - \varepsilon_{XP})$$

or

$$\sigma_X = E(a+by-\alpha T-\varepsilon_{XP}) \tag{4.9}$$

Now this relationship can be substituted into the boundary conditions to produce

$$\int_{-C}^{C} E(a+by-\alpha T-\varepsilon_{XP})dy = 0 \tag{4.10}$$

and

$$\int_{-C}^{C} E(a+by-\alpha T-\varepsilon_{XP})y\,dy = 0 \tag{4.11}$$

For the case in which the properties of the material are considered to be constant, the boundary conditions can be expanded in the following manner

$$\int_{-C}^{C} E(a+by-\alpha T-\varepsilon_{XP})dy = aE\int_{-C}^{C} dy + bE\int_{-C}^{C} ydy - E\int_{-C}^{C} \alpha T dy - E\int_{-C}^{C} \varepsilon_{XP}dy \tag{4.12}$$

and

$$\int_{-C}^{C} E(a+by-\alpha T-\varepsilon_{XP})ydy = aE\int_{-C}^{C} ydy + bE\int_{-C}^{C} y^2dy - E\int_{-C}^{C} \alpha Tydy - E\int_{-C}^{C} \varepsilon_{XP}ydy \qquad (4.13)$$

Since both of these equations are equal to zero, the modulus of elasticity, E, can be divided out. Now if we integrate the equations we will have

$$2ac - \int_{-C}^{C} \alpha dTdy - \int_{-C}^{C} \varepsilon_{XP}dy = 0 \qquad (4.14)$$

and

$$\frac{2}{3}bC^3 - \int_{-C}^{C} \alpha Tydy - \int_{-C}^{C} \varepsilon_{XP}ydy = 0 \qquad (4.15)$$

and now solving for a and b

$$a = \frac{1}{2C}\int_{-C}^{C} \alpha Tdy + \frac{1}{2c}\int_{-C}^{C} \varepsilon_{XP}dy \qquad (4.16)$$

and

$$b = \frac{3}{2C^3}\int_{-C}^{C} \alpha Tydy + \frac{3}{2C^3}\int_{-C}^{C} \varepsilon_{XP}ydy \qquad (4.17)$$

Thus, we have the two constants for the original strain equation. To get the value of strain in the material that is causing stress, the strain caused by free thermal expansion must be subtracted from the total strain. The result is an equation that relates stress directly to strain by the stress-strain curve.

$$\varepsilon = \varepsilon_X - \alpha T = a + by - \alpha T \tag{4.18}$$

$$\varepsilon = \frac{1}{2c}\int_{-C}^{C} \alpha T dy + \frac{3y}{2c^3}\int_{-C}^{C} \alpha T y dy + \frac{1}{2c}\int_{-C}^{C} \varepsilon_{XP} dy \tag{4.19}$$

$$+ \frac{3y}{2c^3}\int_{-C}^{C} \varepsilon_{XP} y dy - \alpha T$$

For any specific metal and temperature distribution, all of the values are known except for plastic strain. By amethod of successive approximations (10), however, the plastic strain can be found from the stress-strain curve.

Procedure for Variable Properties

If the modulus of elasticity is now allowed to vary with temperature, the equations become somewhat more complex. Expanding the boundary equations again, but this time with variable properties, we have

$$a\int_{-C}^{C} E dy + b\int_{-C}^{C} E y dy - \int_{-C}^{C} E\alpha T dy - \int_{-C}^{C} E\varepsilon_{XP}\, dy = 0 \tag{4.20}$$

and

$$a\int_{-C}^{C} E y dy + b\int_{-C}^{C} E y^2 dy - \int_{-C}^{C} E\alpha T y dy - \int_{-C}^{C} E\varepsilon_{XP} y dy = 0 \tag{4.21}$$

Now solving both equations for b and then equating them

$$\frac{\int_{-C}^{C} E\alpha T dy + \int_{-C}^{C} \epsilon_{XP} dy - a \int_{-C}^{C} E dy}{\int_{-C}^{C} E y dy} = \frac{\int_{-C}^{C} E\alpha T y dy + \int_{-C}^{C} E\, \epsilon_{XP}\, y dy - a \int_{-C}^{C} E y dy}{\int_{-C}^{C} E y^2 dy} \tag{4.22}$$

Now solving for the value of a we find

$$a = \frac{\int_{-C}^{C} E\alpha T dy \int_{-C}^{C} E y^2 dy + \int_{-C}^{C} E\, \epsilon_{XP} dy \int_{-C}^{C} E y dy}{\int_{-C}^{C} E dy \int_{-C}^{C} E y^2 dy - (\int_{-C}^{C} E y dy)^2} \tag{4.23}$$

$$+ \frac{-\int_{-C}^{C} E\alpha T y dy \int_{-C}^{C} E y dy - \int_{-C}^{C} E\, \epsilon_{XP} y dy \int_{-C}^{C} E y dy}{\int_{-C}^{C} E dy \int_{-C}^{C} E y^2 dy - (\int_{-C}^{C} E y dy)^2}$$

$$b = \frac{-\int_{-C}^{C} E\alpha T dy \int_{-C}^{C} E y dy - \int_{-C}^{C} E\, \epsilon_{XP} dy \int_{-C}^{C} E y dy}{\int_{-C}^{C} E dy \int_{-C}^{C} E y^2 dy - (\int_{-C}^{C} E y dy)^2} \tag{4.24}$$

$$+ \frac{\int_{-C}^{C} E\alpha T y dy \int_{-C}^{C} E dy + \int_{-C}^{C} E\epsilon_{XP} y dy \int_{-C}^{C} E y dy}{\int_{-C}^{C} E dy \int_{-C}^{C} E y^2 dy - (\int_{-C}^{C} E y dy)^2}$$

Just as before, the values of a and b are substituted into the strain equation to find the strain and thus the stress at any point. The successive approximation method mentioned before is once again used. This method consists of first solving the equations by letting the plastic strain in the integrals be zero. This will give a value for the total strain at a point, then taking the stress-strain curve for that point a new value of plastic strain can be found. (See Figure 5.) This new plastic strain value is now substituted into the plastic strain integrals of the equations and a new value for plastic strain is found as before. The process is repeated until the desired convergence of successive approximations is achieved. Of course, since the modulus of elasticity, yield point, and ultimate strength vary with temperature, the appropriate stress-strain curve must be used for each point at which the stress is to be found.

The question may arise as to the assurance that this method of successive approximations will converge or if it is possible that each successive approximation becomes progressively worse with the result being that the solutions are meaningless. It can be shown that an integral equation of the form

$$y(x) = y_o \int_{x_o}^{x} f\{t,\omega(t)\}dt \qquad (4.25)$$

involving the dependent variable under the integral sign (which is of the same basic form as the integral for a and b) and the function $y(x)$ regarded as the unknown, may be solved by a method of successive

approximations (16). This means that for the case of the thin plate a solution must result if a large enough number of iterations are performed. But this does not say anything about the rate of convergence, which may be slow. With the use of high-speed computers, the problem of slow convergency may be of little concern, but in any case a smaller number of iteractions can be achieved by the use of a good initial estimate for the amount of plastic strain. Thus, after the first calculation of stress is made, an initial guess for the plastic strain of the previous plastic strain found, in most cases, will lead to more rapid convergence.

Modifications for Residual Strains

So far, we have thought of the plate as being free of any previous plastic strains with each new stress calculation. This assumption is quite valid as long as the temperature gradient is increasing, that is the temperature difference between two points on the plate is becoming larger, for if this is the case, each new calculation will produce a larger total strain and a larger plastic strain. But if the temperature gradient is decreasing, each new stress calculation will result with not only a smaller total strain, but also a smaller plastic strain, which would be incorrect.

If a metal is strained into the plastic region, an amount of plastic set is put into the metal equal to the plastic strain. Then if the total strain is reduced, the previous plastic set will remain; thus the plastic strain will remain the same, but the elastic strain will decrease. This can be seen in Figure 5 where the distance *O-A* is

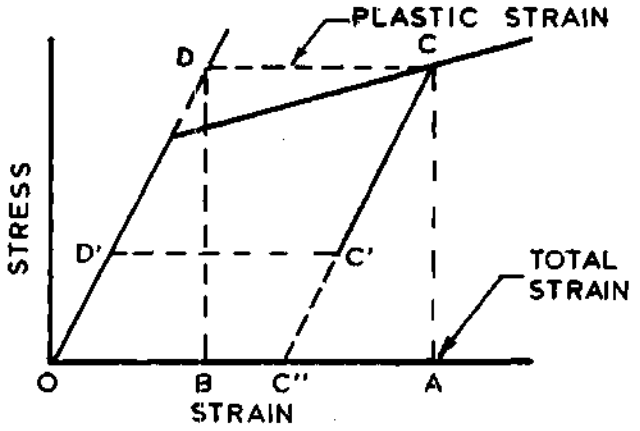

Figure 5. Idealized Stress-Strain Curve with Plastic Set

the initial total strain and B-A (or D-C) is the plastic set. Now if the load is removed the stress-strain relationship will be along line C''-C. But for any point C' along the line C''-C we still have the plastic set of D'-C' or if the load is completely removed, a permanent set of O-C'' which is equal to D-C.

The permanent set that results from plastic flow will produce residual stresses as the material is cooled to a uniform temperature throughout.

The formation of plastic set within a material necessitates the formulation of a method to account for the previous plastic strain. The previous equations written to account for both elastic and plastic strain will apply, but now instead of ε_{XP} the plastic strain will be $\Sigma\varepsilon_{XP} + \Delta\varepsilon_{XP}$ where $\Sigma\varepsilon_{XP}$ is the sum of all of the previous plastic strain and $\Delta\varepsilon_{XP}$ is the new plastic strain that has occurred over the time

interval since the last strain calculation.

The previous equations for strain can be grouped and simplified. For any given temperature distribution, certain integrals in the strains equations will have constant values and if we set these integrals equal to a constant we have:

$$EQ1 = \int_{-C}^{C} Ey^2 dy \tag{4.26}$$

$$EQ2 = \int_{-C}^{C} Eydy \tag{4.27}$$

$$EQ3 = \int_{-C}^{C} Edy \tag{4.28}$$

$$DIV = \int_{-C}^{C} Edy \int_{-C}^{C} Ey^2 dy - \left(\int_{-C}^{C} Eydy\right)^2 \tag{4.29}$$

or

$$DIV = (EQ3 \cdot EQ1) - (EQ2)^2 \tag{4.30}$$

$$B1 = \int_{-C}^{C} E\alpha Tydy \tag{4.31}$$

$$B2 = \int_{-C}^{C} E\alpha Tdy \tag{4.32}$$

If these constants are now substituted back into the equations for a and b, the equation becomes more compact. If we let $A1 = \frac{EQ1}{DIV}$, $A2 = \frac{EQ2}{DIV}$, and $A3 = \frac{EQ3}{DIV}$ we can write:

$$a = (A1 \cdot B2) + A1 \int_{-C}^{C} E\varepsilon_{XP}dy - (A2 \cdot B1) - A2 \int_{-C}^{C} E\varepsilon_{XP}ydy \qquad (4.23a)$$

$$b = (A2 \cdot B2) - A2 \int_{-C}^{C} E\varepsilon_{XP}dy + (A3 \cdot B1) + A3 \int_{-C}^{C} E\varepsilon_{XP}ydy \qquad (4.24a)$$

But, since we are now considering the previous plastic strain, the integrals for the plastic strain must be expanded

$$\int_{-C}^{C} E\varepsilon_{XP}dy = \int_{-C}^{C} E(\Sigma\varepsilon_{XP})dy + \int_{-C}^{C} E(\Delta\varepsilon_{XP})dy \qquad (4.33)$$

$$\int_{-C}^{C} E\varepsilon_{XP}ydy = \int_{-C}^{C} E(\Sigma\varepsilon_{XP})ydy + \int_{-C}^{C} E(\Delta\varepsilon_{XP})ydy \qquad (4.34)$$

In these integrals the one with the $\Sigma\varepsilon_{XP}$ term is the total previous plastic strain, while the one with the $\Delta\varepsilon_{XP}$ term is the integral which is iterated. Using this procedure the equation for the strain at any point must be modified so that the plastic strain read from the stress-strain curve is only the additional plastic strain ($\Delta\varepsilon_{XP}$) and not the total plastic strain ($\Sigma\varepsilon_{XP} + \Delta\varepsilon_{XP}$).

With this modification, the equation for strain becomes

$$\varepsilon = a + by - \alpha T - \Sigma\varepsilon_{XP} \qquad (4.35)$$

The effect of this change is the same as that of moving the stress-strain curve to the right by an amount equal to the previous plastic strain.

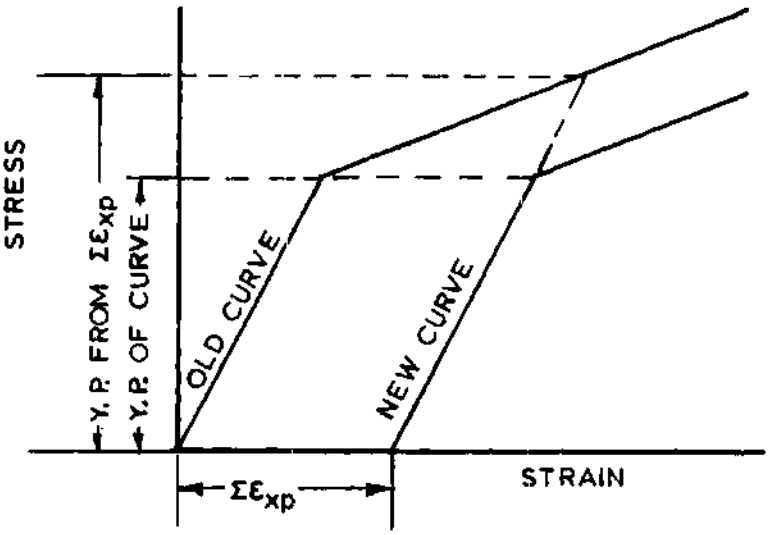

Figure 6. Transposed Stress-Strain Curve

This effective change can be seen in Figure 6 above.

Modification for Changes in Yield Strength

The new stress-strain curve, shown in Figure 5 on page 35, has not taken into account the change in the yield point which is caused by cold working. In order to account for this increase in yield point, a new yield point must be calculated for the new stress-strain curve. This calculation can readily be done for idealized stress-strain curve, through the use of geometric relations.

Shown in Figure 7 on the following page is an idealized stress-strain curve. Point *A* is the original yield point of the material and point *B* is the new yield point after the material has been loaded to point *B* and then the load removed. The original strain at yielding is ε_{YPV}; the new yield strain after work harding is ε_{YP}, the Modulus of

Elasticity is *E* and the slope of the plastic part of the curve is *K*.

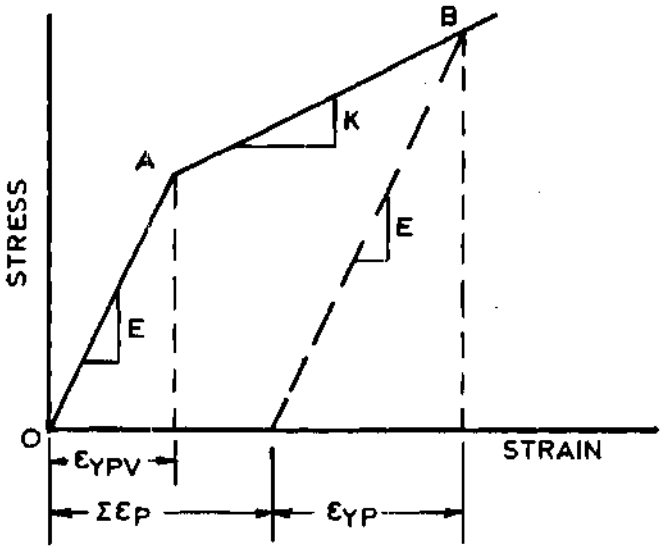

Figure 7. Idealized Stress-Strain Curve

It is possible to express the stress for any point in terms of strains and the curve slopes:

$$\sigma = K\varepsilon - K\varepsilon_{YPV} + E\varepsilon_{YPV}$$

or

$$\sigma = K\varepsilon + (E-K)\varepsilon_{YPV} \qquad (4.36)$$

If the stress is now written for point *B* in this manner, it would be:

$$\sigma_B = K(\Sigma\varepsilon_P + \varepsilon_{YP}) + (E-K)\varepsilon_{YPV} \qquad (4.37)$$

The new yield point strain can be found by dividing the stress at point *B* by the modulus of elasticity.

$$\varepsilon_{YP} = \frac{K}{E}\Sigma\varepsilon_P + \frac{K}{E}\varepsilon_{YP} + (1 - \frac{K}{E})\varepsilon_{YPV} \tag{4.38}$$

Now collecting terms and solving for the new yield point, we have:

$$(1 - \frac{K}{E})\varepsilon_{YP} = \frac{K}{E}\Sigma\varepsilon_P + (1 - \frac{K}{E})\varepsilon_{YPV} \tag{4.39}$$

$$\varepsilon_{YP} = (\frac{K}{E-K})\Sigma\varepsilon_P + \varepsilon_{YPY} \tag{4.40}$$

If the material were now loaded in tension until yielding and then loaded in compression until yielding by an equal amount so that in the final unloaded state there would be no residual strains, the above formula would indicate that the yield point would be the same as that of virgin material. This would mean that the work hardening caused by compression cancelled out that of tension, but this is not correct for actually they would tend to be additive. This would require that the sum of the plastic strain be rewritten as the sum of the absolute values of the plastic strains.

$$\varepsilon_{YP} = (\frac{K}{E-K})\Sigma|\varepsilon_P| + \varepsilon_{YPV} \tag{4.41}$$

In order to convert this to the yield point stress, it is only necessary to multiply the new yield point strain by the modulus of elasticity.

$$\sigma_Y = E\,\epsilon_{YP} = \left(\frac{EK}{E-K}\right)\Sigma|\epsilon_p| + E\,\epsilon_{YPV} \qquad (4.42)$$

Although this method should provide a reasonably accurate value for the yield point, it does not include all effects; for instance, the Bauschinger effect (2) where plastic flow in one direction will tend to reduce the yield point stress at which flow occurs in the opposite direction. This is not considered nor is any allowance made for stress relaxation or creep that could occur at high temperature.

Another problem that is encountered is the determination of the yield point for a transient temperature gradient, as the temperature of the material changes so does the stress-strain curve. This means that the yield point is constantly changing; thus making it difficult to define a yield point after plastic flow has occurred, but by use of the previous formula and the parameters from the proper stress-strain curve an appropriate value can be calculated.

Conclusion

With the theory and equations presented in this chapter and the use of an appropriate data on modulus of elasticity, yield point, ultimate strength, and coefficient of thermal expansion, the stress can be calculated for any point across a plate.

CHAPTER V

ANALYTICAL SOLUTION

The Computer Program

Now, using the methods explained in Chapters III and IV for calculating the temperature gradient and the stresses in a flat plate, a computer program was written for use on the Burroughs 5500 Computer. In essence, this program (see Appendix) first makes a temperature calculation to determine the temperature at which the phase transformation occurs, next, using this data it recalculates the transient temperatures and, finally, from the temperature gradient, the strains and stresses are calculated.

Transformation Section

Sections one and two of the program are almost identical in operation. In section one (phase change temperature calculations) the transient temperature and the time at which they occur are determined using variable properties, except for total enthalpy. In Figure 3 it can be seen that at high temperatures the total enthalpy curve is almost linear. If this section of the curve were replaced by a straight line and extended into the low temperature range, it would eliminate the effect of a phase change. Thus, if the linearized total enthalpy curve is used for the temperature calculation, the resulting temperature can be used to determine the phase change points. As has already been mentioned in Chapter III, the phase change will occur as the transient

temperature crosses the T-T-T curve.

In order to incorporate a T-T-T curve into a computer program, the curve must be approximated by either a series of equations or by some numerical method. The fitting of an equation or equations to a T-T-T curve could be quite complicated and a good fit may not result. The method that was used was basically numerical in that the T-T-T curve was approximated by a series of steps as shown in Figure 8. This approximation can easily be programmed and the accuracy of the fit can be made extremely good if the time increment is made sufficiently small (large number of steps).

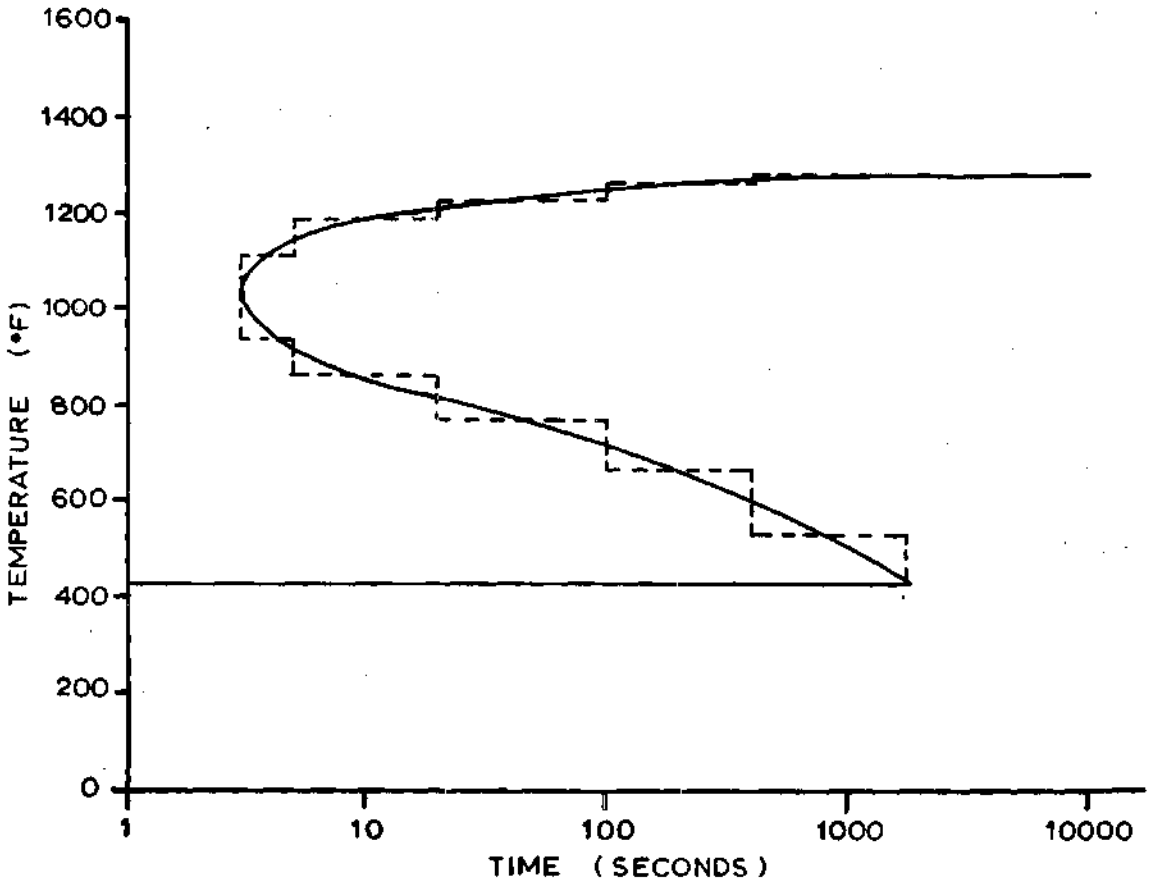

Figure 8. Approximated T-T-T Curves

Temperature Section

The second section which makes the actual transient temperature calculation is executed in the same way as section one, but this time the total enthalpy curves are used. Since the phase transformation temperature will vary across the material each point for which a calculation is made will have a different total enthalpy curve. In Chapter III it was stated that a good approximation of the proper enthalpy curve for any phase change could be produced by moving the original curve along line A-A in Figure 3. As cited earlier in this chapter, curves can be entered into a computer by either of two methods: as an equation or as a set of numbers; again, for the same reasons as before, the numerical approach was used. The total enthalpy curve, as are all data curves, was read in as set of point taken at specified temperature intervals. Then, in order to find the value of the curve in any temperature interval, a linear interpolation was done between the end points. Two equations were used to shift the total enthalpy curve to its proper position for different phase transformation temperature. The shift along the x or temperature axis was accomplished by the equation

$$T'_N = T_N - (T_P - T'_P) \qquad (5.1)$$

where T'_N is the new temperature axis location, T_N is the old temperature axis location, T_P is the phase change temperature for the enthalpy curve, and T'_P is the temperature at which the phase change occurred for that

point in the metal. The enthalpy shift was accomplished by a similar equation

$$I_N' = I_N - (S \times (T_P - T_P')) \qquad (5.2)$$

where I_N' is the new enthalpy point on the Y axis, I_N is the old enthalpy point, S is the slope of the axis of inversion (line A-A in Figure 3) and T_P and T_P' are the same temperature as before. It is noted that the values of temperature and enthalpy on the axis are subscripted. This means that points with the same subscript are the same point on the curve. Thus, with the curve transposed, the temperature calculations can be completed.

Stress Section

The calculation of thermal stress was discussed in Chapter IV and there is no need to repeat it here. The reading into the computer of data curves was accomplished by the numerical means already mentioned with the transposing of the thermal expansion curve performed by the same equation used to move the total enthalpy curve, except that in the equations enthalpy is replaced by thermal expansion. The variation of the coefficient of expansion with temperature is shown in Figure 9.

By use of an idealized stress-strain curve, the problem of having a stress-strain curve for every temperature was greatly eased. Curves for modulus of elasticity, yield point stress, ultimate stress and ultimate strain all versing temperature were read into the computer and then from this data the proper curve for any given temperature was

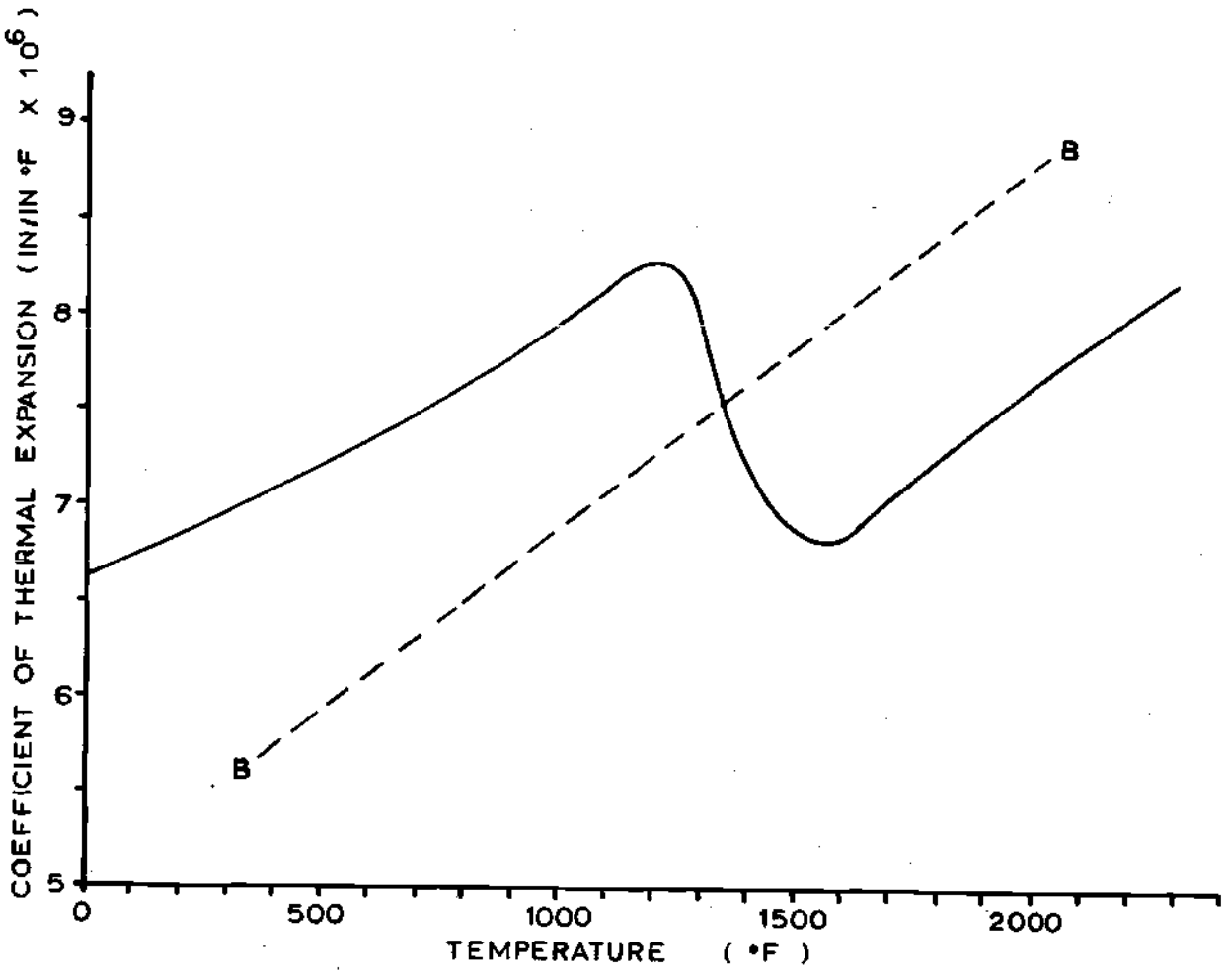

Figure 9. Variation of the Coefficient of Thermal Expansion with Temperature for AISI C1020 Steel

constructed. These data are shown in Figures 10, 11, and 12. Also, since the curve is constructed within the program any change in the yield strength that occur from work hardening are also easily accounted for. If work hardening takes place, the ultimate strain is decreased by an amount equal to the previous plastic strain ($\Sigma\varepsilon_p$) and the ultimate strength is modified so that the slope of the curve in the plastic region (K) is the same after the change in yield strength as it was in the original curve.

Method of Integration

In sections one and two, the calculation of the temperature gradient is strictly a numerical procedure, but the calculation of the strains as presented in Chapter IV is based on a set of integrals. Since the digital computer cannot directly solve integrals, it was necessary to rely on a numerical method. Any of a number of numerical methods of integration could be used (17), but for the computer program Simpson's Rule was used which can be written as

$$\int_{x_0}^{x_2} f(x)dx = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]. \qquad (5.3)$$

This method was chosen over the Trapezoidal Rule in that it was of about the same complexity, yet would provide a more acurate solution. Also, while some of the more involved methods may give a more accurate answer, they place requirements upon the number of points integrated over, while Simpson's Rule requires only that there be an odd number of points.

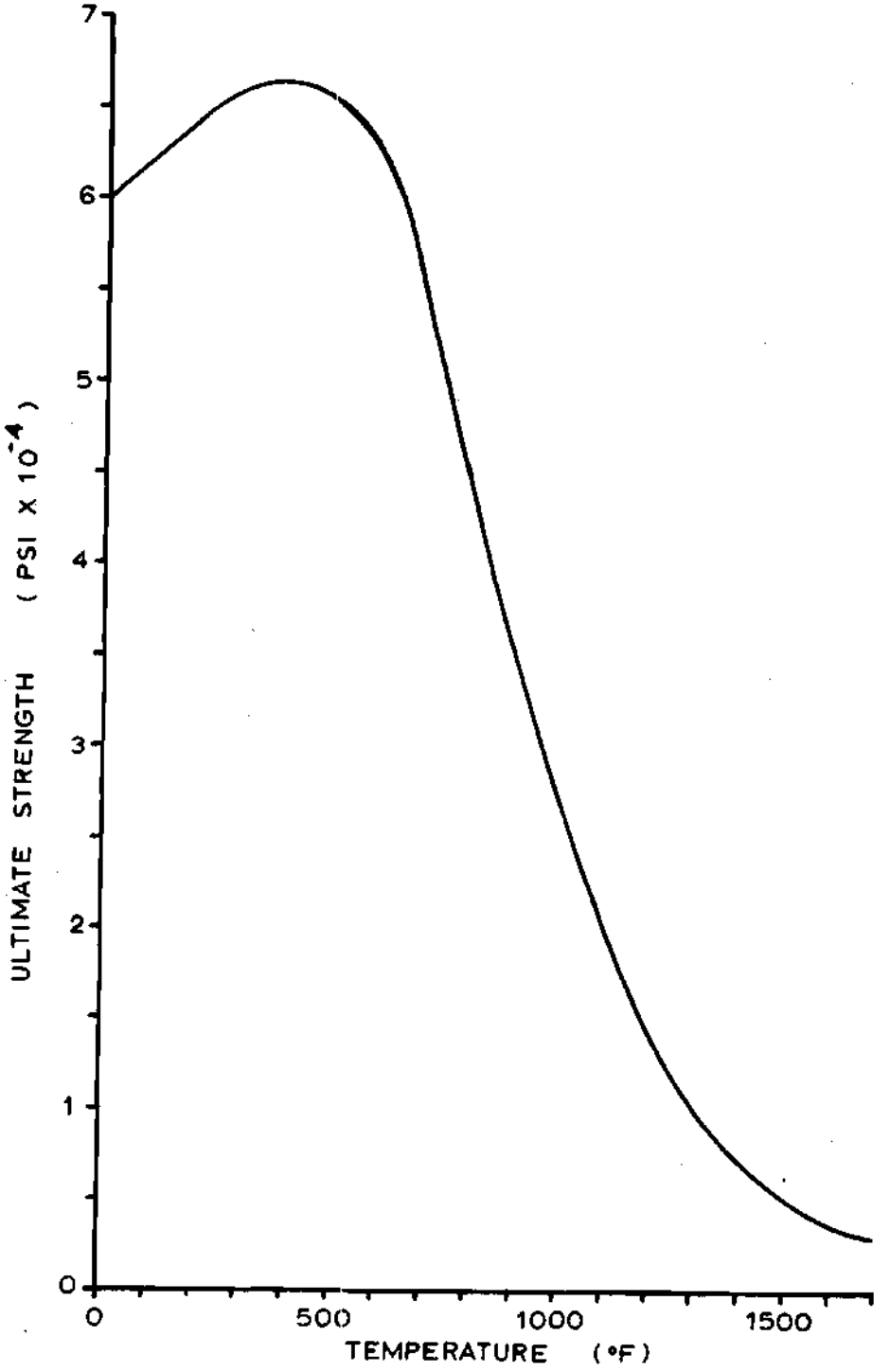

Figure 10. Variation of the Ultimate Strength of AISI C1020 Steel with Temperature

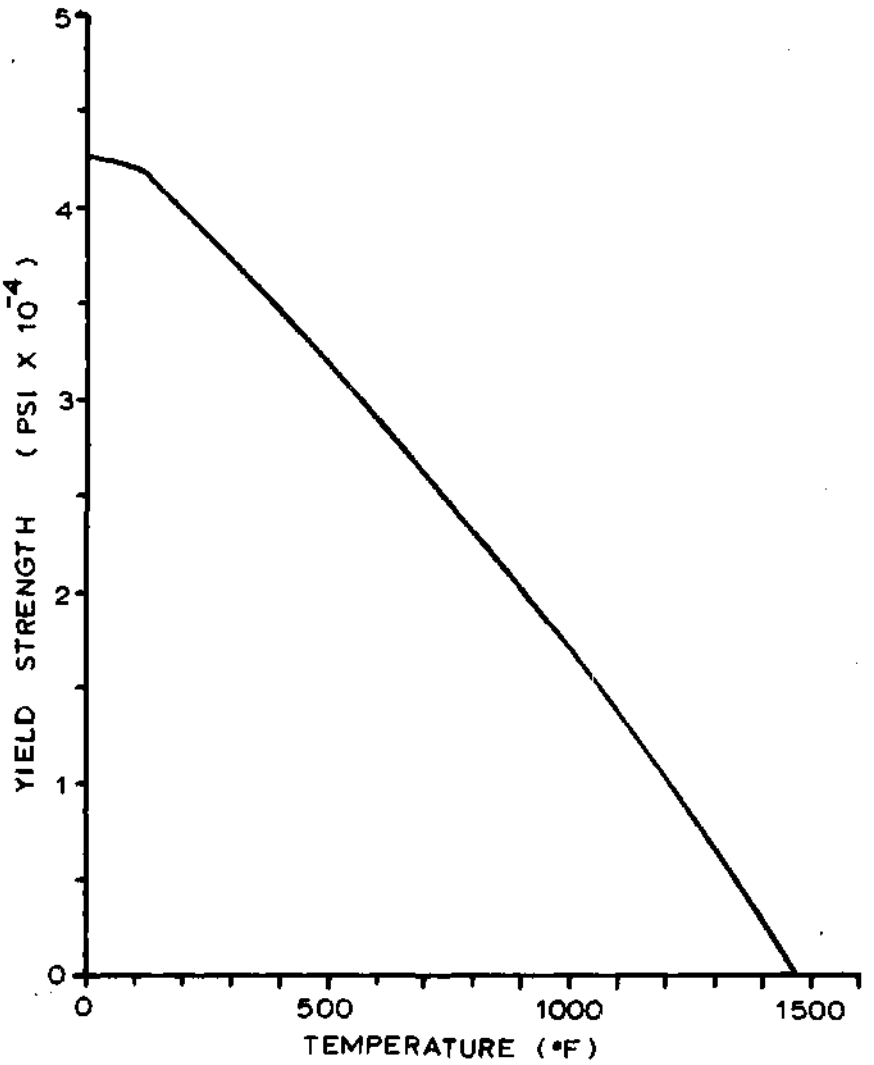

Figure 11. Variation of the Yield Strength of AISI C1020 Steel with Temperature

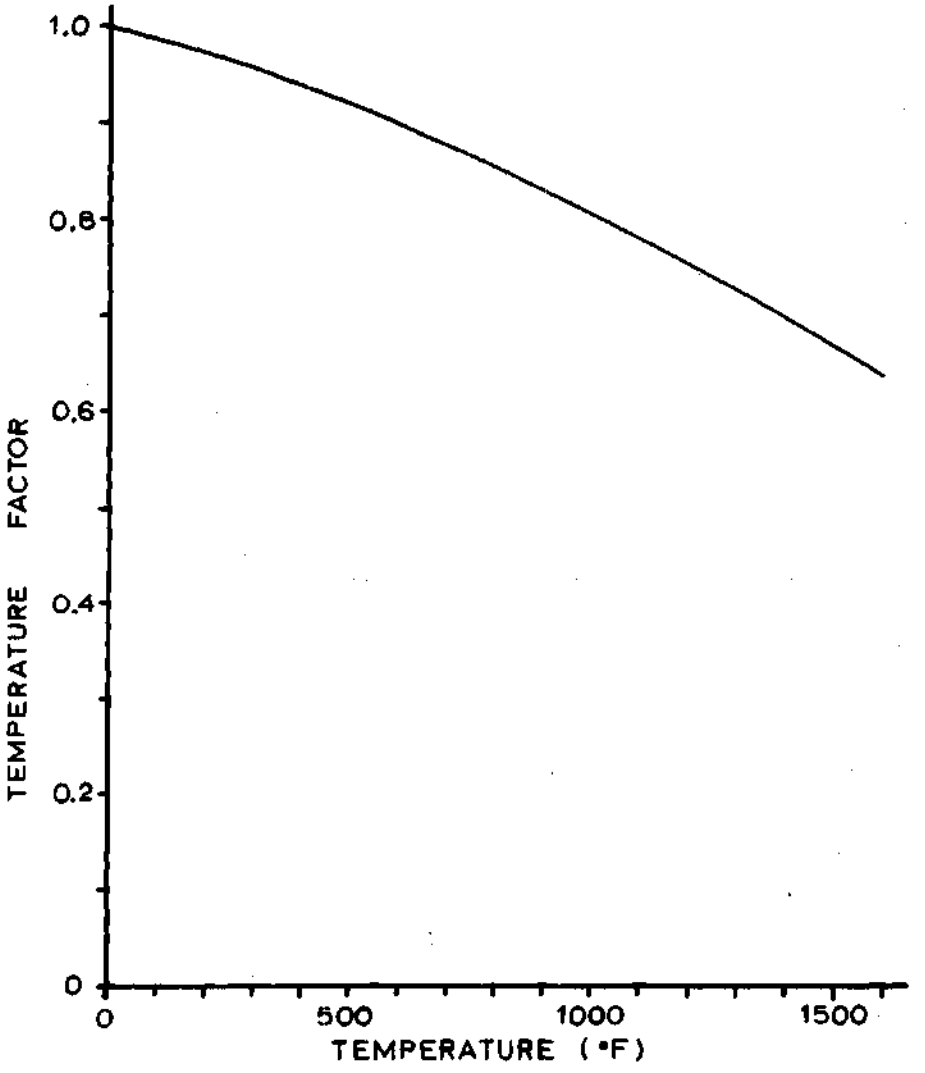

Figure 12. Temperature Factor Curve for Young's Modulus at High Temperatures

CHAPTER VI

COMPARISON OF RESULTS

Cases Studied

Because of the general way in which the computer program was written, it was possible to run data for many different cases and situations. In order that a comparison may be made of the effects of making the calculation under various assumptions, six cases were run. For each of the six cases the initial and boundary conditions were maintained at the same values. In this manner the properties of the material would be the only thing that would affect the stress value. Also, since the temperature distribution would have an effect upon the stress, the temperature for all cases was calculated by using the same variable properties so that each case would be calculated from the same temperature distribution.

The six cases considered were:

Case One. The properties of the material (AISI 1020 Steel) were variable with temperature and the effect of phase transformation at temperatures other than 1330°F was considered. Also, the metal was allowed to flow plastically. Since this case considers more of the properties to be variable, it will be assumed to be most accurate.

Case Two. The properties again varied with temperature and the calculations were made with plastic flow, but this time phase change occurred at 1330°F. That is, no allowances were made for rapid cooling.

Case Three. Again the properties varied with temperature, but this time the material was considered to be all elastic.

Case Four. The properties of the material for the stress calculation were those of the material at 800°F and were assumed to be constant at this value for all temperatures. The calculations were made with plastic flow.

Case Five. The properties are the constants at 800°F, but this time the material was considered to be elastic only.

Case Six. This case is the same as Case Four, except that now the properties were the values at 70°F.

In all of the six cases calculated, the initial temperature of the plate was 1500°F with an ambient temperature of 100°F. The initial temperature of 1500°F was chosen so that the plate will be cooled from a temperature that was high enough for a phase change to occur. A surface or film coefficient of 1000 BTU/hr-Sq.ft°F was used because this value would be approximately that of using water to cool the plate, a rapid cooling process.

Sources of Data

Before it is possible to make any calculations, it is necessary to have information about the properties of the material to be studied. For an accurate comparison to be made of the cases, it is necessary that this information be correct.

The properties of metals at normal temperatures is easy to find, but the properties for temperatures up to 1600°F can present some problems. There were four prime sources of data used in the six cases

calculated: *British Iron and Steel Research Association Physical Constants of Some Commercial Steels* (14), "The Ferrous Metals Book" (18), "Note on the Temperature Variation of Young's Modulus of Various Steels" (19), and *ASM Metals Handbook* (13). The book *British Iron and Steel Research Association Physical Constants of Some Commercial Steels* contains data on total heats, mean specific heats, coefficients of thermal expansion, and thermal conductivities for plane carbon steels, and low alloy and high alloy steels. Since this is a British publication, all the units are metric, but by use of the proper conversion factor they can be changed to American units. The second source mentioned, "The Ferrous Metals Book," presents physical properties, mechanical properties, elevated temperature properties, and low-temperature properties for most carbon steels. This book also presents data for resulphurized steels, low-alloy steels, corrosion resistant steels, heat resistant steels and stainless steels.

In the article "Note on the Temperature Variation of Young's Modulus of Various Steels," F. W. Jones and J. Nortcliffe present what they call a temperature factor. This temperature factor relates the Young's Modulus at room temperature to the Young's Modulus at higher temperatures (up to about 1500°F). They state that from a set of constants (temperature factors) the modulus for practically all ferritic steels can be calculated. Thus, this article provides a means for finding Young's Modulus at high temperatures for carbon steels.

The *Metals Handbook* Provides a wealth of information about steel, but the main datum used from it was the transformation-temperature-time

curve for low-carbon steel. While the T-T-T curves were the only information used, the book has data on the properties of both ferrous and nonferrous metals.

Effects of Phase Change

Cases One and Two differ only by the fact that for Case One modifications had been made for phase change; thus, it can be assumed that any differences in the resulting stresses for the two cases can be attributed to phase change. It can be seen in Figure 14 that the stress at the edge in Case One increases during the first ten seconds to about 10,000 PSI, then decreases to about 3,000 PSI, and then once again begins to increase, while the stress for Case Two increases to about 20,000 PSI, and then decreases.

From the stress equations in Chapter IV, it can be seen that the stress at any one point does not only depend upon the strain and physical properties at that point, but is influenced by the state of strain and the properties of all the other point of the plate. Figure 13 is a plot of the temperature distribution and phase transformation temperatures. This graph will thus indicate the section of the plate that has undergone phase transformation, any area that is below the transformation curve and above the temperature curve has changed phase. It can be seen then that a section about 0.12 inches from the outside edge is transforming after about four seconds. Since phase transform is not instantaneous, the actual change begins then at some time before four seconds.

In Figure 9 it can be seen that before the phase change begins the coefficient of thermal expansion is decreasing during the phase change it is increasing, and after the transformation it is decreasing again. This means that during the phase transformation the material in this section that is undergoing the transformation will expand (see Figure 24). This increase is the increase in negative strain at this location, and thus causes an increase in compressive stresses. This effect can be seen in the stress distribution around 0.12 inches for Case One after three and five seconds (Figure 16).

The increase in the compressive stresses of this interior zone has the net effect of reducing the tensile stresses on the outside edge as can be seen in Figure 14. As the plate continues to cool, this section cools beyond the transformation and again the coefficient of expansion begins to decrease. This decrease reduces the compressive stresses at the 0.12-inch point as can be seen in the 10-second curve in Figure 16, also there is an increase in the tensile stresses along the edge at this time (see Figure 14).

After 15 seconds all but the outside 0.05 inches has been transformed with the result being that the outside edge for Case One has a smaller compressive stress level than Case Two. By studying Figures 13, 16, and 17 it can be seen that the area for Case One which has not undergone phase transformation tends to have a smaller tensile stress (or a larger compressive stress) than do the same areas in Case Two which has already undergone the phase transformation. Figure 13 would indicate that after about 20 to 25 seconds all phase transformation for Case One

has taken place and that the state of stress for both Cases One and Two would be the same, and this is approximately true as shown by the stress distribution curves after 30 seconds in Figures 16 and 17. The slight differences in these two curves might be accounted for by the fact that phase transformation did not occur in the same manner for the two cases. For Case One, the phase change began at an interior zone then moved inward to the center and after all of the center section had been transformed, the zone moved outward to the edges, while for Case 2 the outside edges were the first to transform with the zone moving inward so that the center was the last to transform.

It is shown in Figures 22 and 23 that the residual stresses and strains that result by making the calculations with the properties considered as in Cases One and Two are almost identical. Thus, considering phase change in the calculation of residual stresses and strains has little effect, while for the calculation of transient stresses the effect may be significant. It would appear that the use of phase transformations in the calculation of transient stresses will result in larger tensile stresses and smaller compressive stresses over those calculated by using variable properties only.

Effect of Plastic Flow

Case Three was calculated to examine the compatibility of an all-elastic solution with one that considers plastic flow. Of course, this to a great extent depends upon the amount of plastic flow, but for the example studied the elastic solution appears to be almost useless. By looking only at the stress of the outside edge of the plate, it can be

seen (Figure 14) that stresses calculated as if it were all elastic are about 4-1/2 times larger than the tensile stresses calculated under the same conditions, but with plastic flow accounted for, and about two times larger than the compressive stresses. Also, an all-elastic solution shows only tensile stresses on the outside and compressive stresses in the center, while a plastic solution shows both tension and compressive at the edges and center over the period of cooling.

An interesting facet of the results for Case Three is that while no plastic flow was considered the properties of the material were allowed to vary with temperature with the normal phase transformation occurring at about 1330°F, the result of this phase change was a variation from the expected stresses. Usually, for an all-elastic solution, the stress distribution is of the same form as the temperature distribution, but because of the increase in thermal expansion due to a phase change a hump appears in the stress distribution curve (Figure 18). This hump is quite noticeable in the curves for three and five seconds, and it can also be seen that this hump is moving toward the center at the same rate as the phase transformation is moving. After ten seconds of cooling, all phase transformation has been completed and as a result the hump has disappeared from the stress distribution, and the curve is the expected image of the temperature distribution curve. Also as would be expected since there are no residual strains, there are no residual stresses.

Effects of Constant Properties

Cases Four and Six were calculated to show the difference that would result from the use of constant properties. Case Four used the properties of AISI C1020 Steel at 800°F and Case Six was for 70°F. On examining the stress distributions for Case Four (Figure 19) and Case Six (Figure 21), it can be seen that the shape of the curves is about the same for both cases, but that the scale for Case Six is twice that of Case Four. Thus, any discussion that is valid for Case Four is equally valid for Case Six and vice versa. By comparing the stresses as the edge (Figures 14 and 15), it would seem that the results obtained by using constant properties and those obtained by using variable properties agree favorably if the curve for Case Four is used up to nine seconds and the curve for Case Six is used from that point on. But these curves are misleading, for if the stress distribution curves for variable properties (Figure 17) are compared with those for constant properties (Figures 19 and 21), it can be seen that the stresses across the plate for the constant property case are much larger than those stresses for the variable property case. At about 30 seconds, though, the reverse is true and the variable property case has the larger stresses.

Residual Stresses and Strains

The residual stresses and strains calculations provide some interesting information. First, looking at the residual strains (Figure 23) one can see that for the calculations made with variable properties (Cases One and Two), all of the plate underwent plastic strain while the

constant property calculations (Cases Four and Six) show no plastic deformation in sections that are at a distance of about one quarter of the width from the edges. Now looking at the residual stress curves for Cases Four and Six, it is seen that these sections are in tension, this would indicate that the over-all length of the plate after cooling is longer than that of the plate before heating and cooling. The residual strain curves for the calculation made with variable properties (Cases One and Two) show that all of the residual strains are negative; thus indicating the over-all length of the plate decreases. Thus, while all of the previous results have shown some form of agreement the final residual strains show a contradiction. This reduction in length that is found by using variable properties can be accounted for by looking at the first few seconds of cooling. At the beginning of cooling, the temperature of the plate is high and as a result the yield point is almost zero, as the plate is cooling the temperature of the outside drops drastically with an increase in yield strength, while the inside has a very low yield strength. The cooling produces tensile stress on the outside and compressive stress inside, and since the inside in very weak, it yields extensively in compression. This large compressive plastic set that is formed almost instantaneously is of such magnitude that any following plastic flow is insufficient to overcome its effect. Thus, if the residual strains (and over-all size) are of importance a great deal of difference can result by making the calculations with constant properties instead of variable properties.

Suggestion for Future Study

Throughout this discussion it has been assumed that the results obtained for Case One are the most accurate, as this case takes into consideration more variables. But this assumption may not be true and the stresses found from constant properties may be just as correct as those found from variable properties. This leads to the need of experimental work to verify the accuracy of the results obtained in Case One. A simple experiment that could be conducted to verify the result that after cooling all the residual strains are negative and thus the size of the material has decreased would be to heat a piece of AISI C1020 Steel to 1500°F and then quench cooling to room temperature. If the dimensions of the specimen are measured before heating, and then again after cooling any decrease in the dimension would verify the results obtained with variable properties and an increase would mean that the results obtained with constant properties may be more accurate.

Also, the work presented here could be extended to two-dimensional and three-dimensional solutions. Then using these solutions a comparison could be made of several cases as was done here.

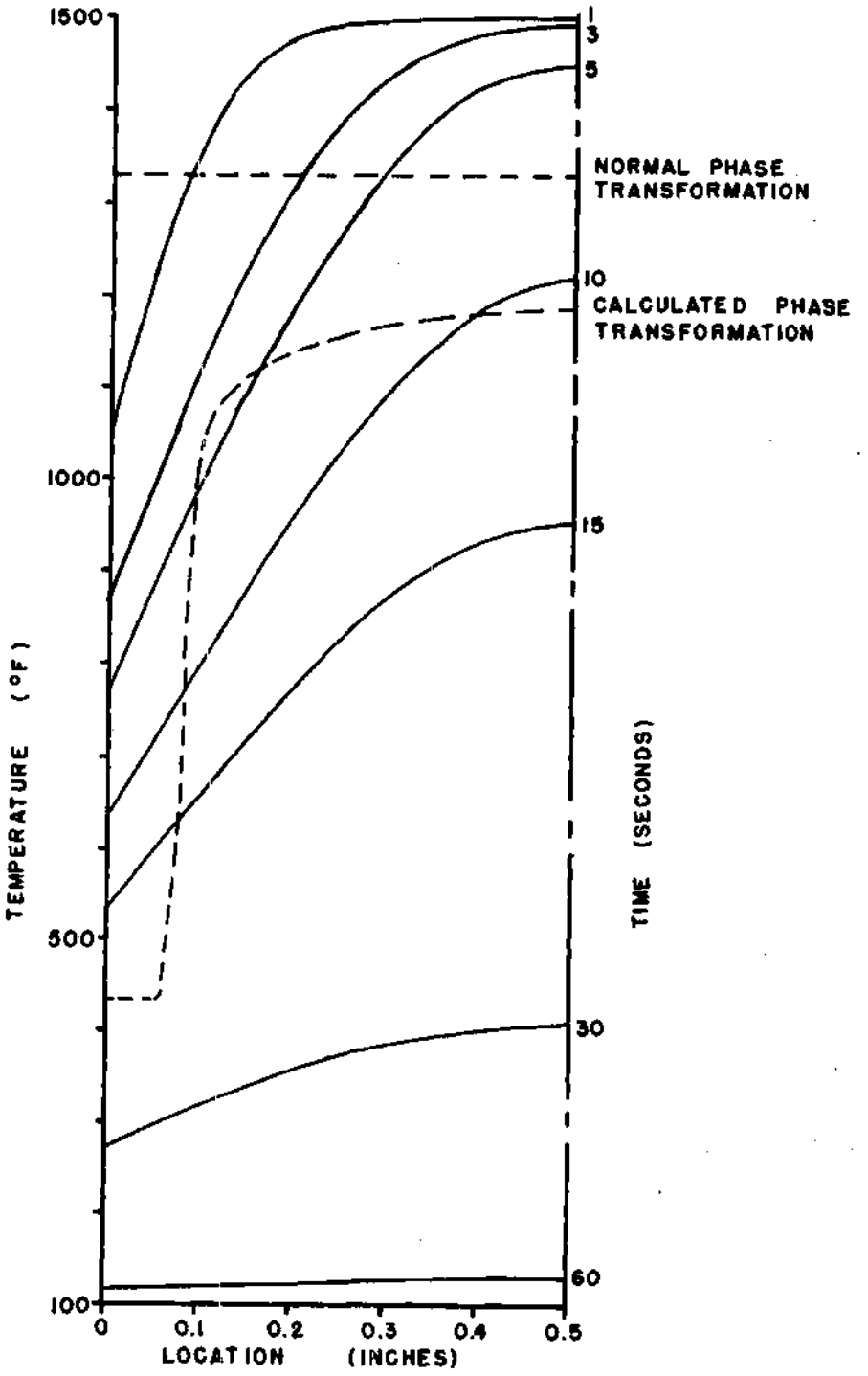

Figure 13. Temperature Distribution at Various Times and Phase Transformation Temperatures

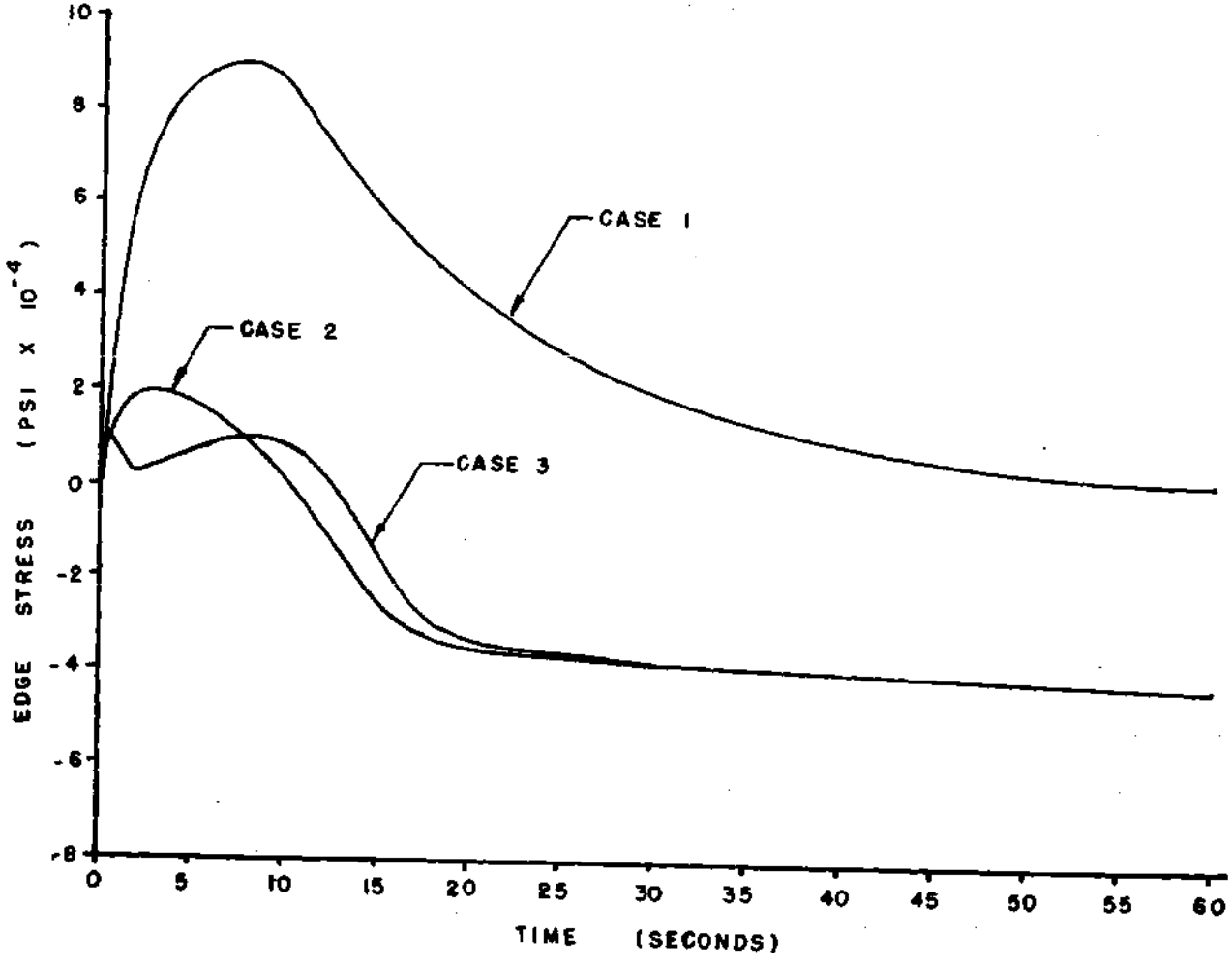

Figure 14. Edge Stresses for Cases One, Two and Three

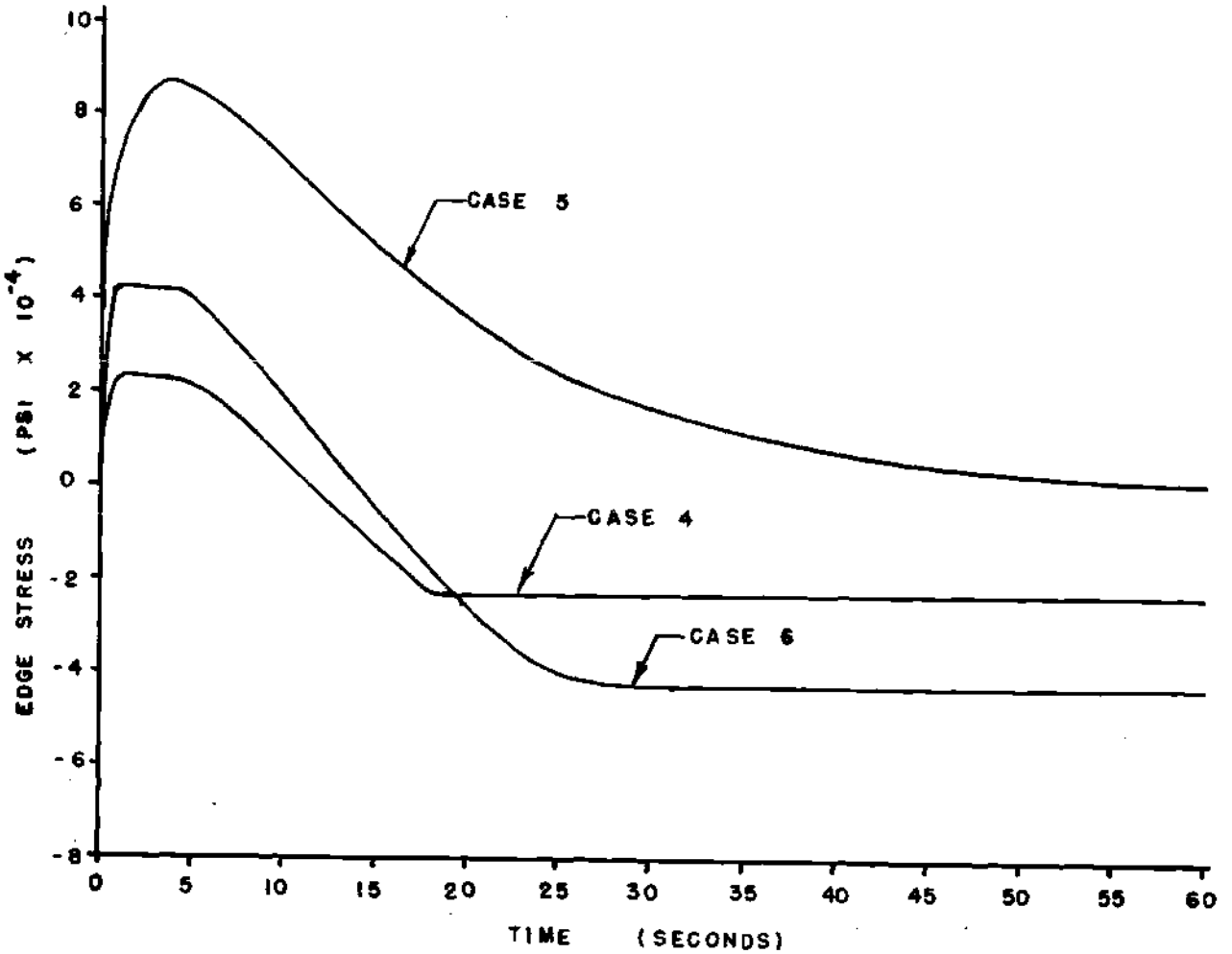

Figure 15. Edge Stresses for Cases Four, Five, and Six

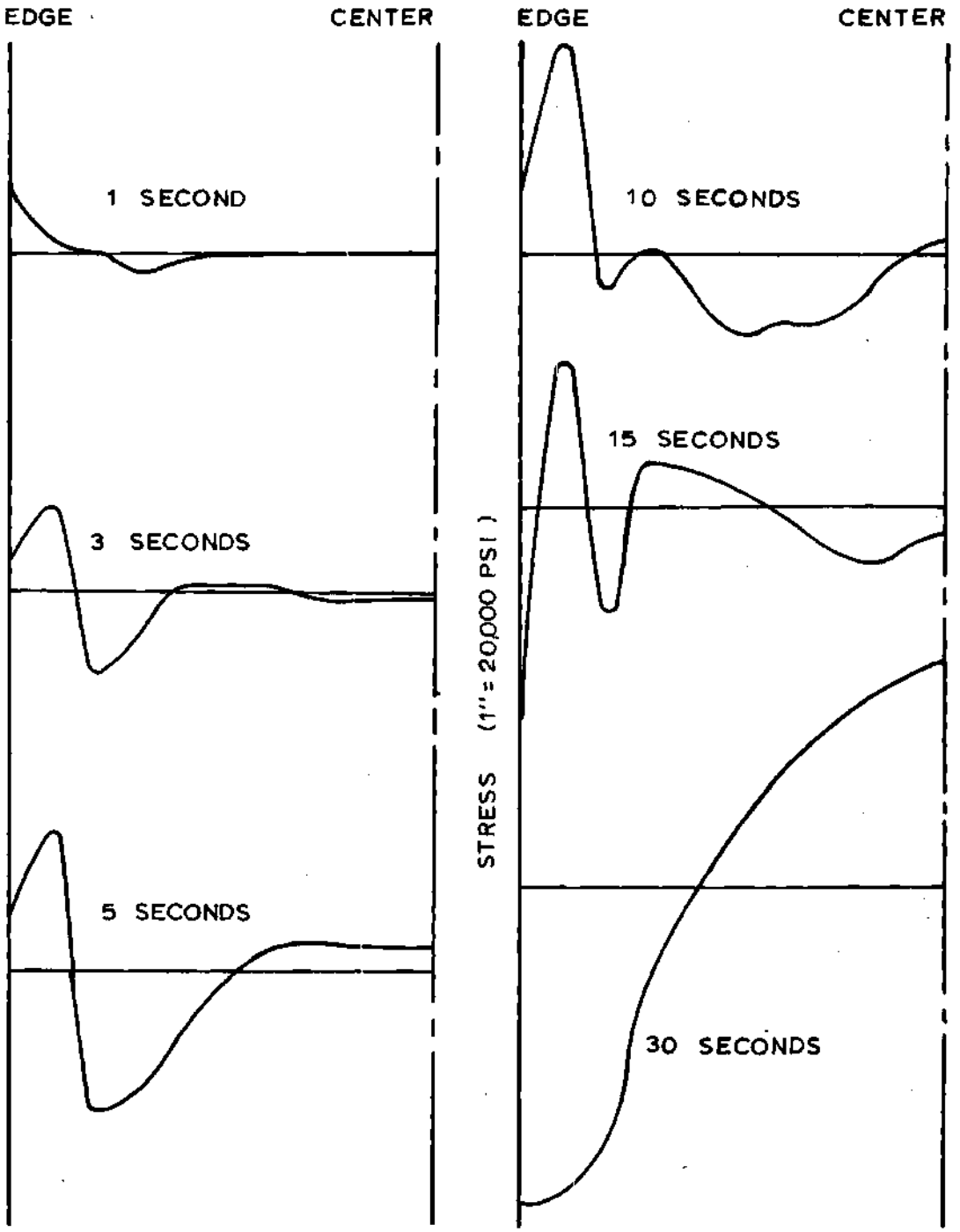

Figure 16. Stress Distribution at Various Times, Case One

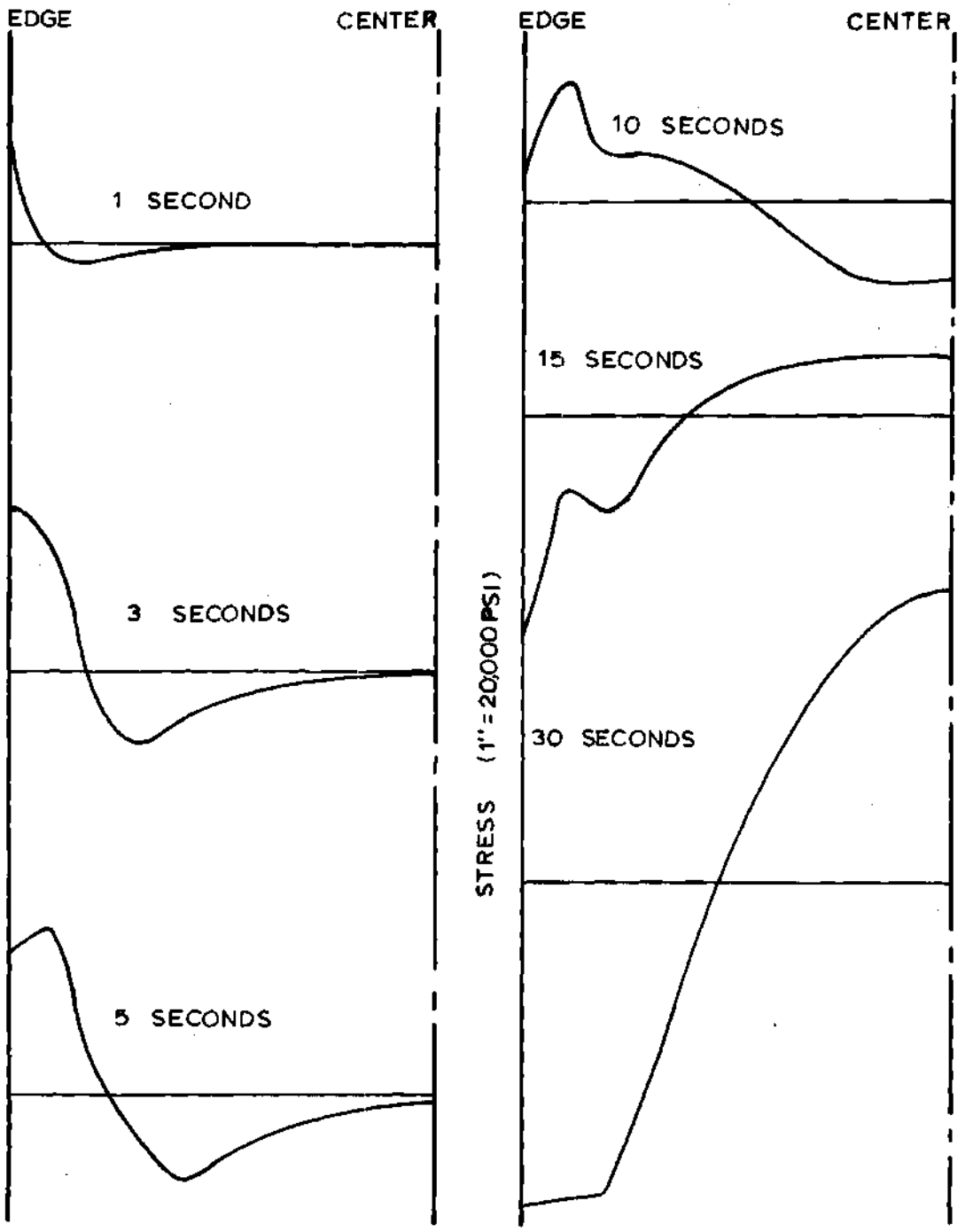

Figure 17. Stress Distribution at Various Times, Case Two

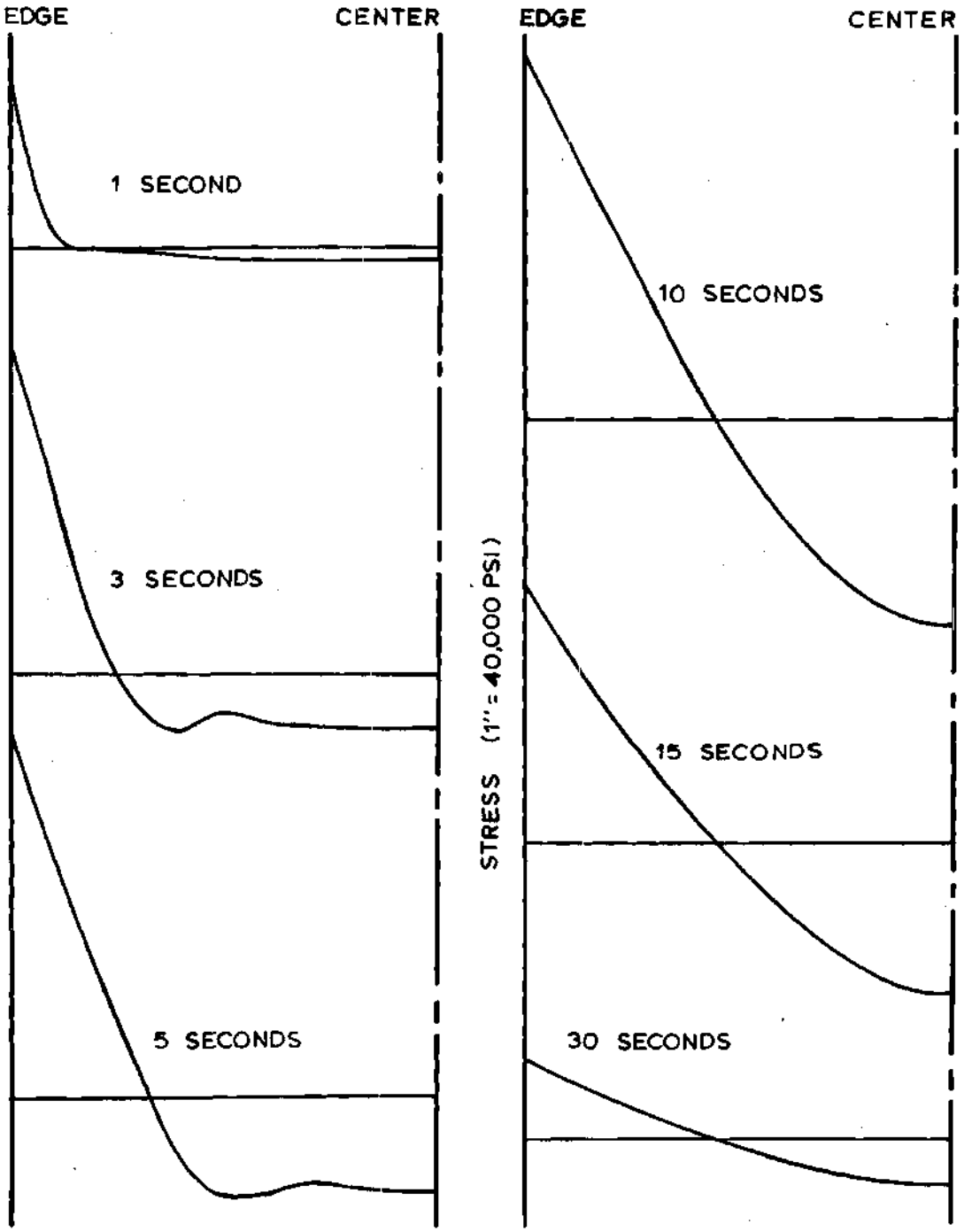

Figure 18. Stress Distribution at Various Times, Case Three

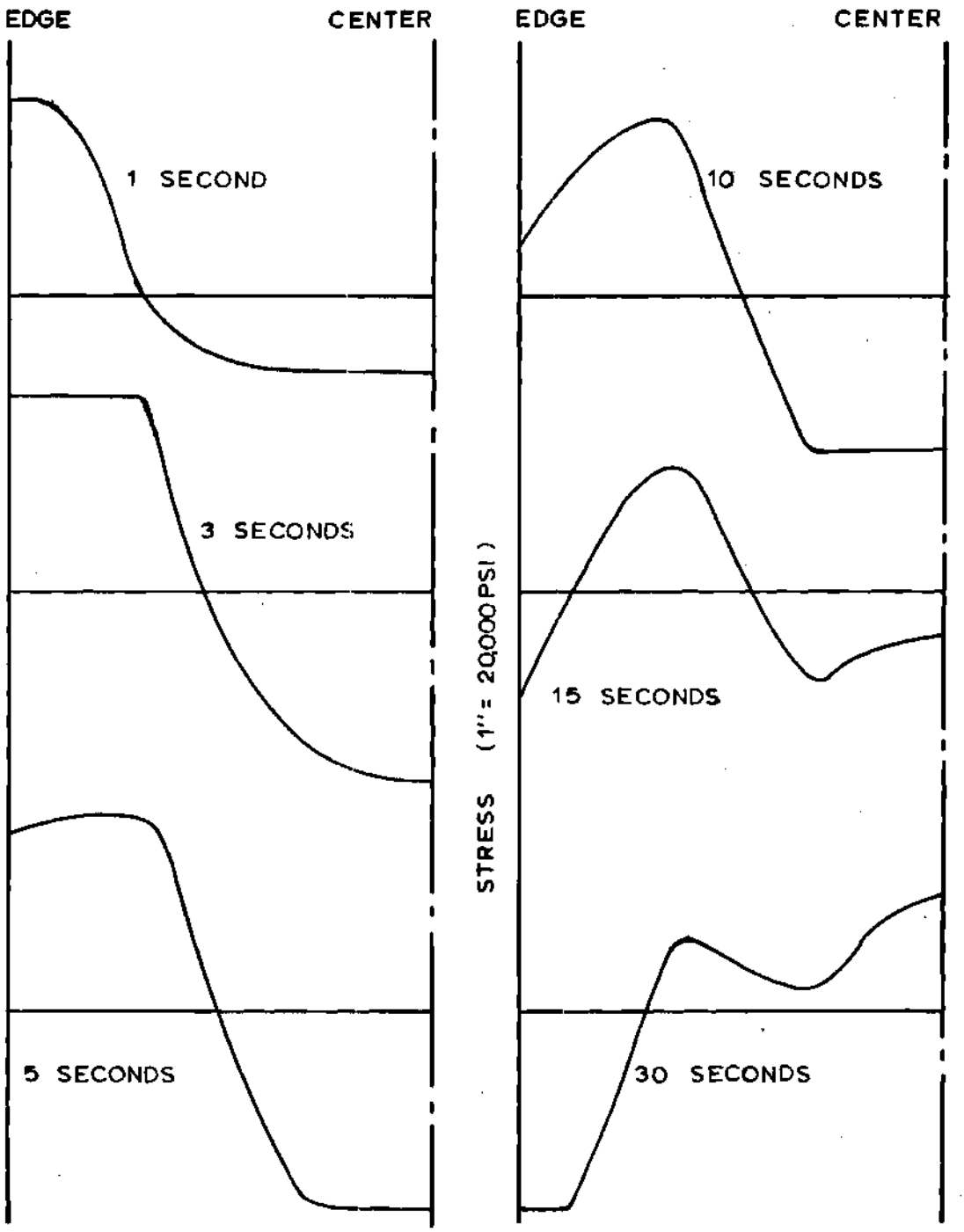

Figure 19. Stress Distribution at Various Times, Case Four

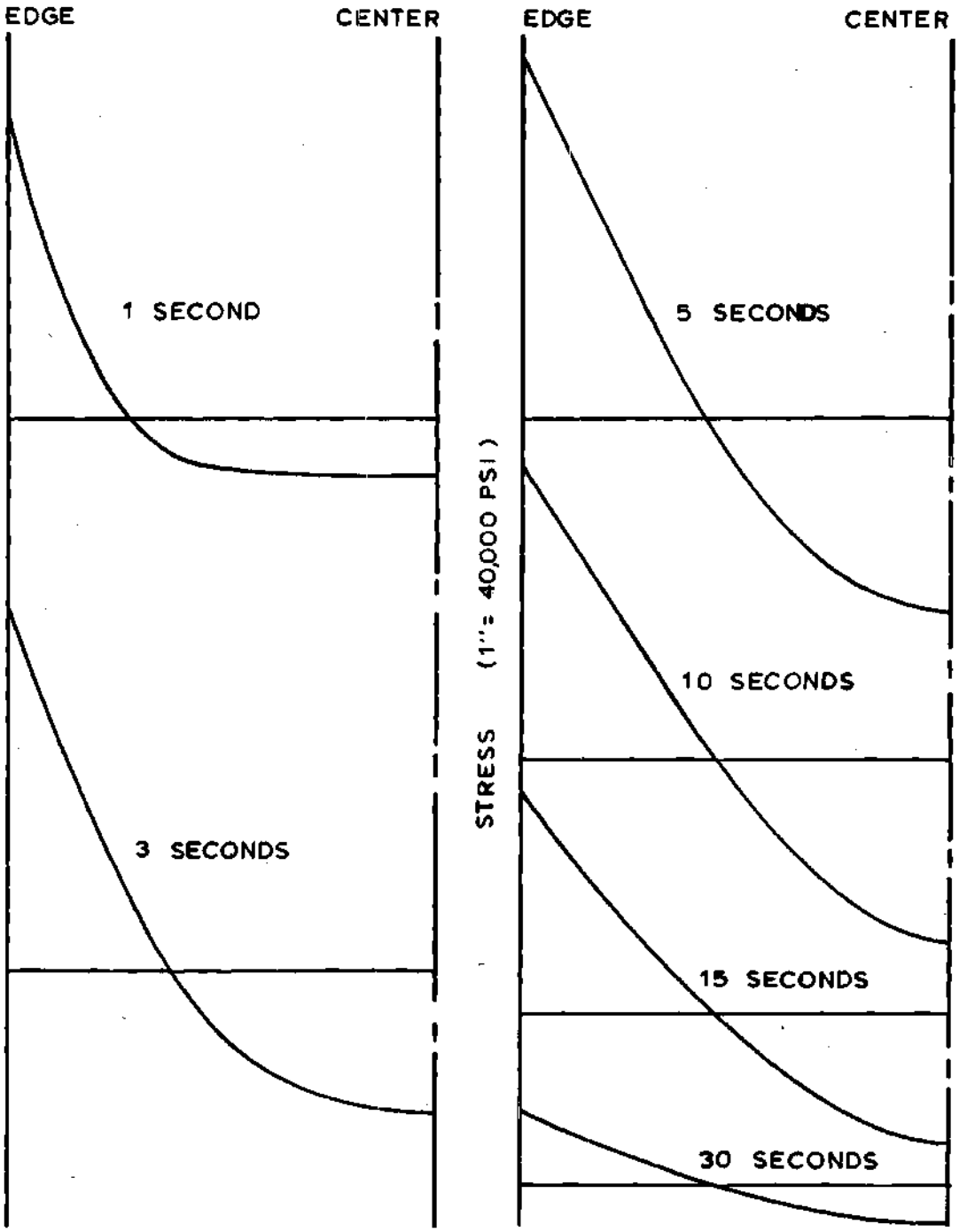

Figure 20. Stress Distribution at Various Times, Case Five

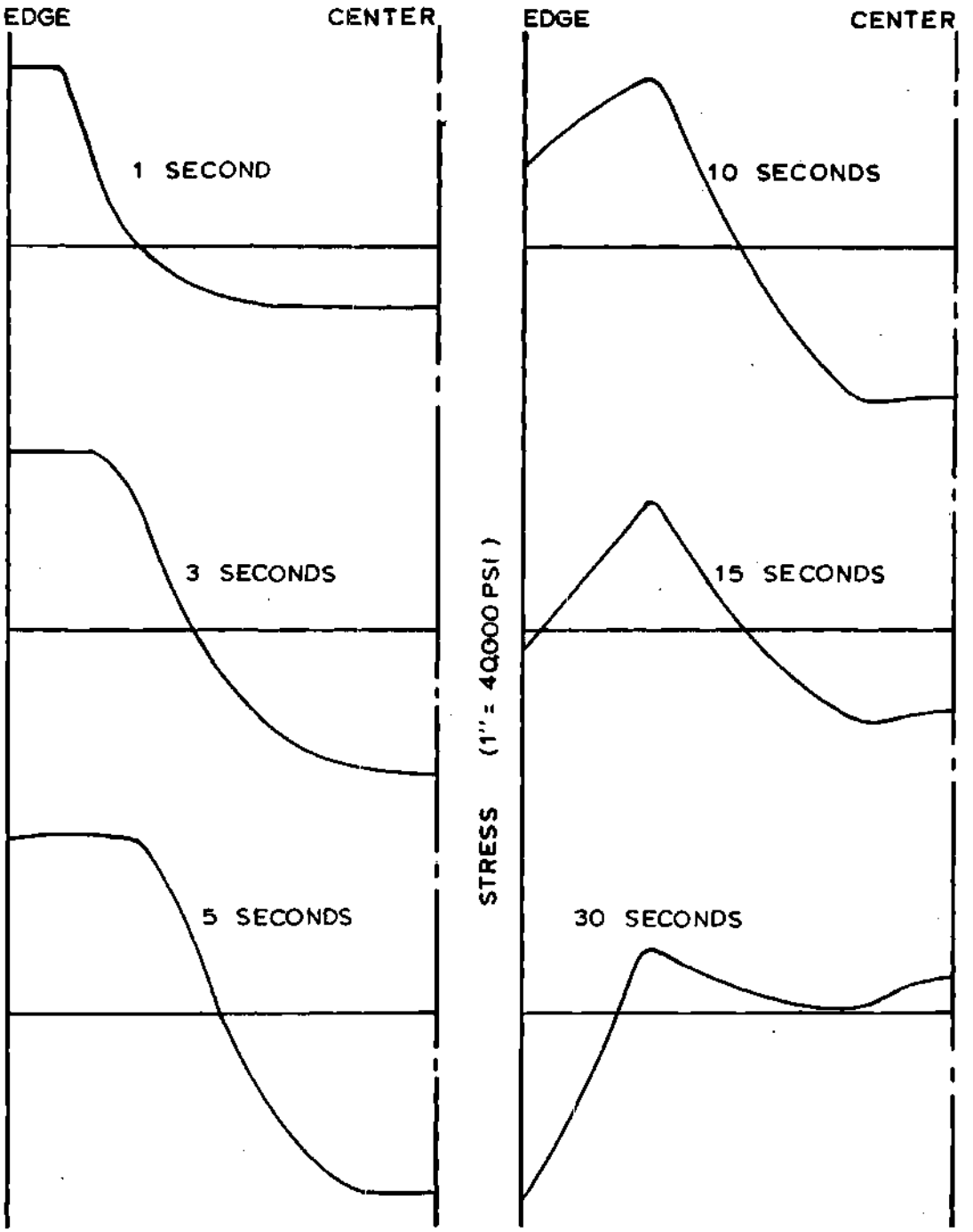

Figure 21. Stress Distribution at Various Times, Case Six

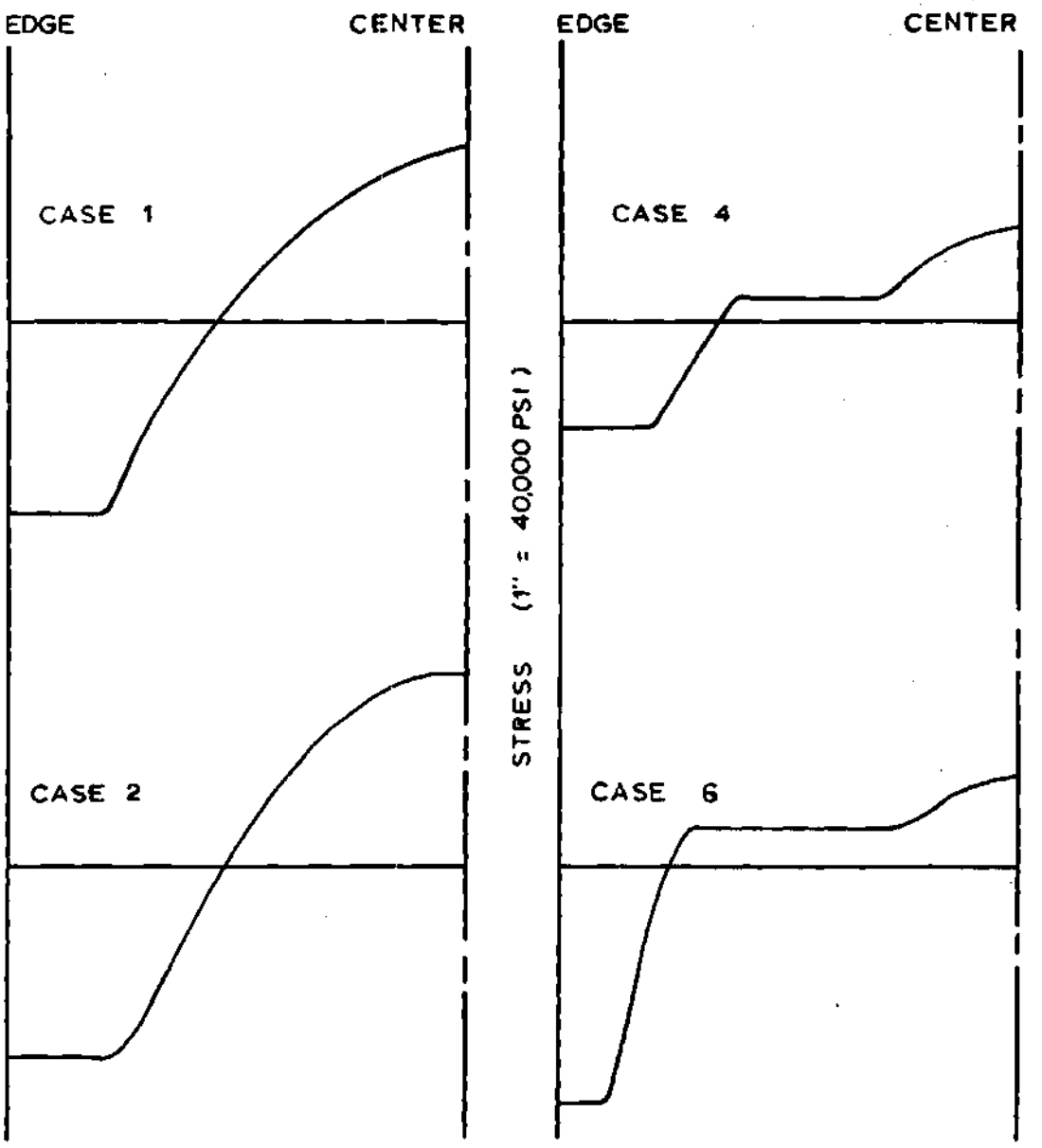

Figure 22. Residual Stresses

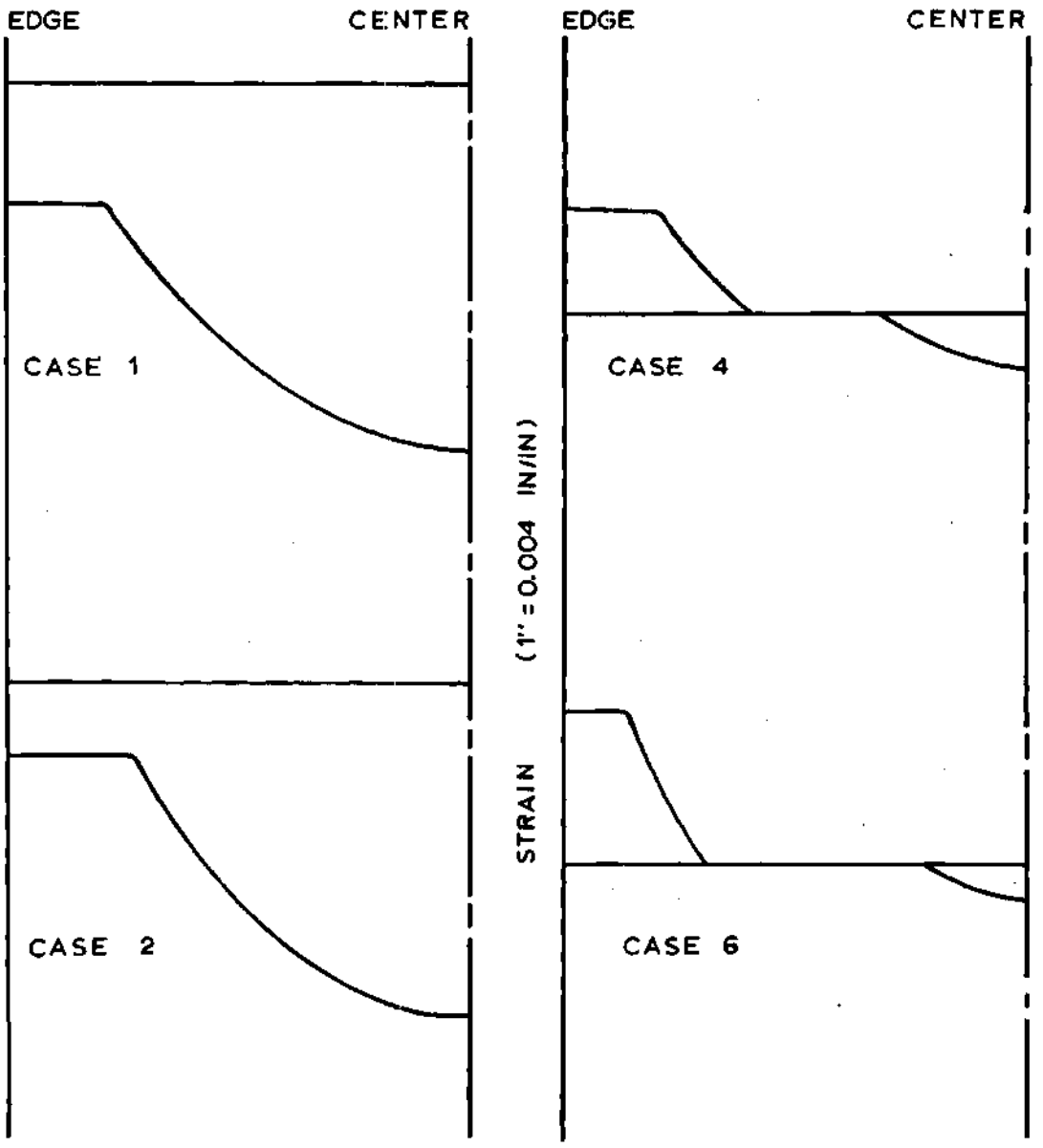

Figure 23. Residual Strains

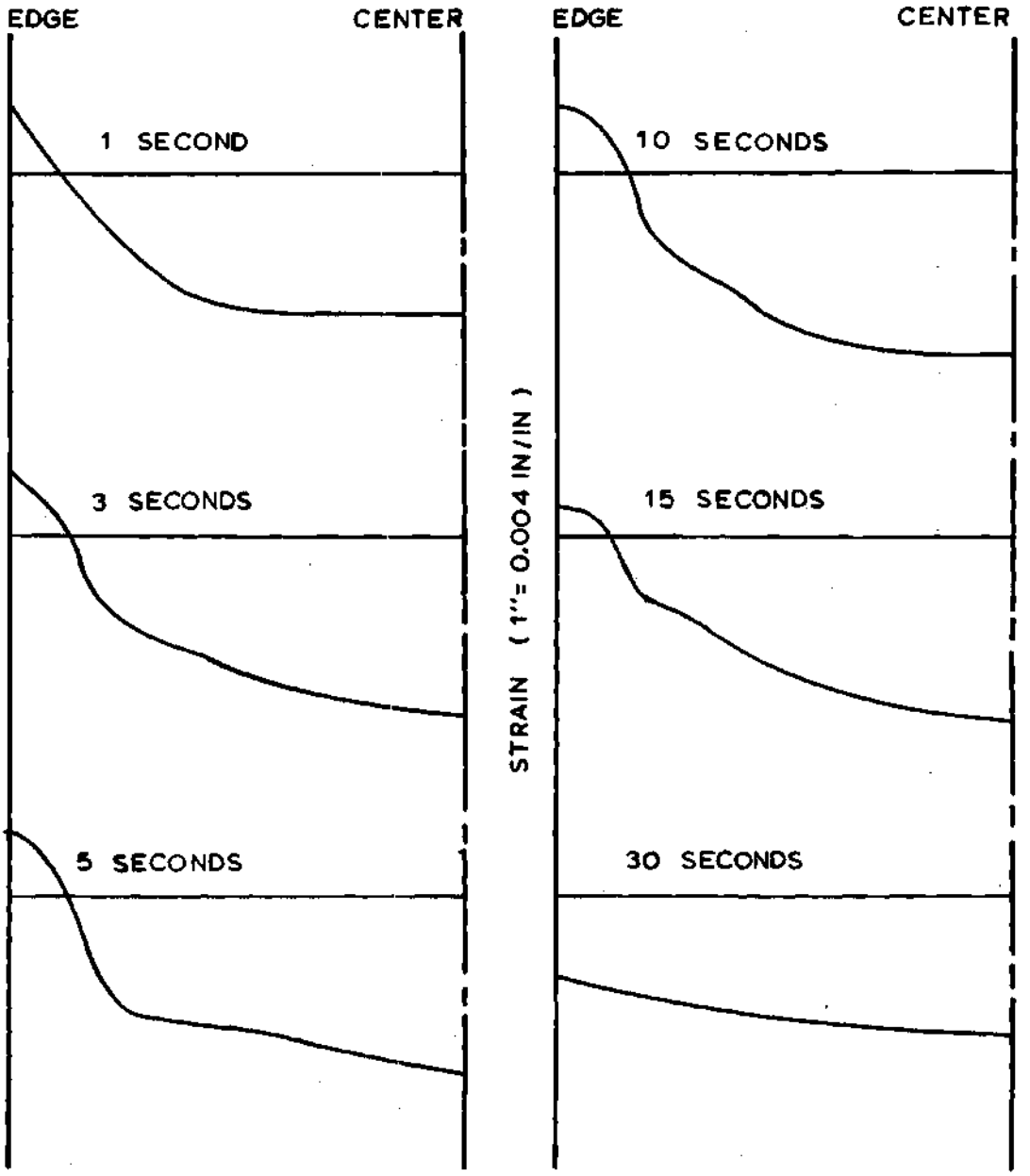

Figure 24. Strain Distribution at Various Times, Case One

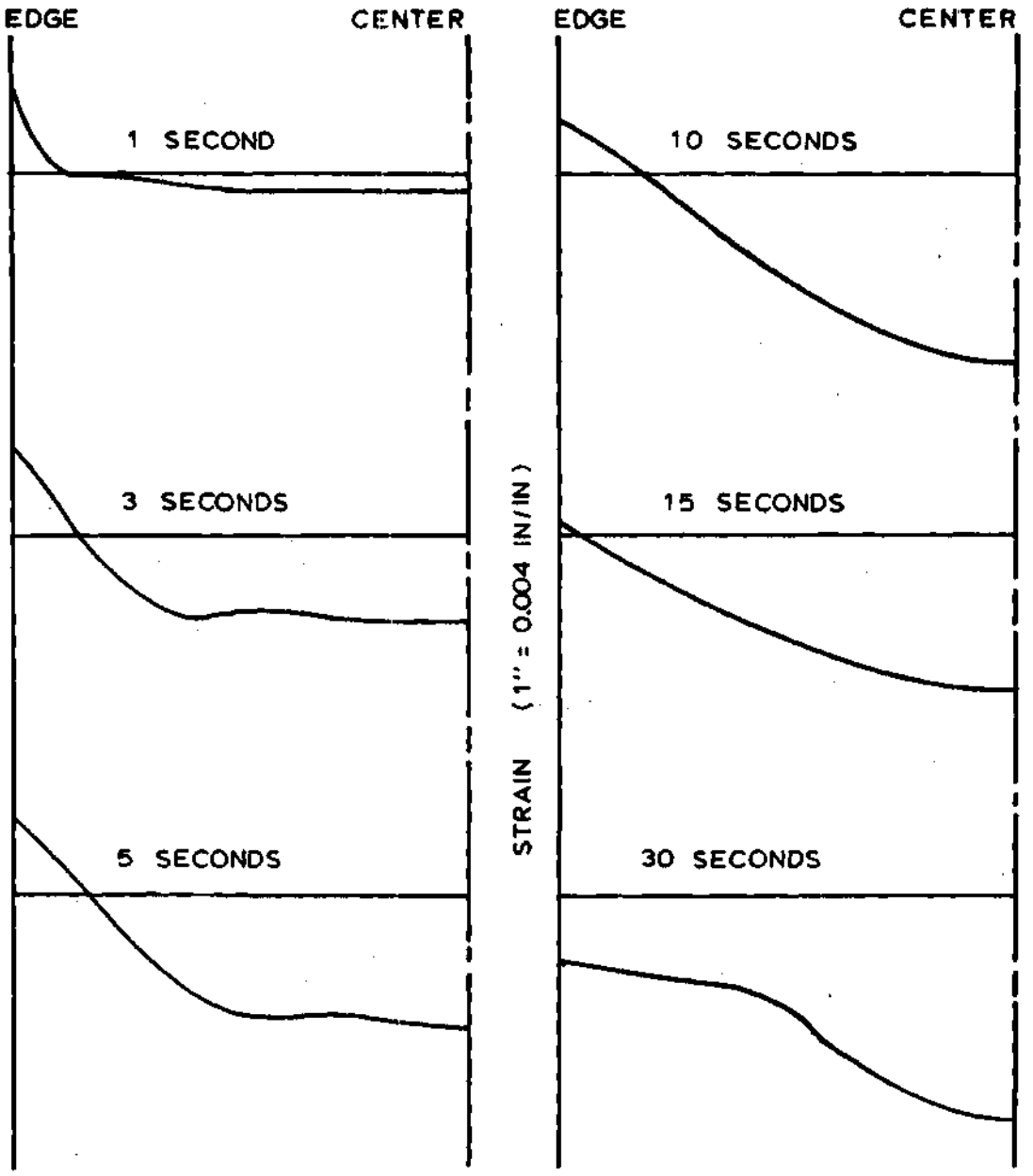

Figure 25. Strain Distributions at Various Times, Case Two

APPENDIX

APPENDIX A

HOW THE PROGRAM WAS RUN

The computer program present here is divided into three sections by comment cards, and a beginning section that contains declaration cards. If a *write card* is added to the beginning section and the first section (the section to calculate phase transform) it is possible to print out the temperature at which phase change will occur. Now a read statement for phase transform (TTRANS[N]°F and TRANS[N]°C) is added to the beginning section and the last two sections the stresses can be calculated. If for the problem considered the phase transformation temperature distribution is known or if the transformation temperature is considered to be constant the above procedure will result in a saving computer time. It was in this manner the computer program was run to obtain the data for the cases studied. Also, since the program contains a large number of array the size of each array was reduced to that required by the data and the *real array* were changed to *save array* so that they would be stored in the computer and thus reduce input-output time.

The above modifications were performed on the program so that time could be saved in obtaining results for the six cases, but these modifications are not necessary to the running of the program.

APPENDIX B

HOW TO READ DATA INTO THE COMPUTER

The list below is the manner in which the data is read into the computer. The beginning of each new line is the beginning of a new card. Since free field read is used, there is no requirement as to how the data is placed on a card except a comma must follow each data point.

Area (in^2)
Incremental step (inches)
Temperature of ambient side one (°F)
Temperature of ambient side two (°F)
Initial temperature of the plate (°F)
Density of the material (lb/in^3)
Largest thermal conductivity ($Btu/hr\text{-}ft^2\text{-}F$)
Smallest specific heat (CAL/gm)
Number of thermal conductivity data points
Number of DX's (odd number)
Number of time steps
Surface coefficient of conductivity Side 1 ($Btu/hr\text{-}ft^2\text{-}°F$)
Surface coefficient of conductivity Side 2 ($Btu/hr\text{-}ft^2\text{-}°F$)
Specific enthalpy intercept of Zero°C (cal/gm)
Slope specific enthalpy (CAL/GM/°C)
(Number of time steps X 2)-1
Number of specific enthalpy points
Thermal conductivity points ($CAL/Cm^2/cm/°C/SEC$)
Time interval points from T-T-T Curve (Sec)
Temperature at interval points lower (°F)
Temperature at interval points upper (°F)
Specific enthalpy points on 50°C inter 1 (Cal/gm)
Modulus of elasticity at room temperature (PSI)
Slope thermal expansion (in/in/°F)
Number of data points to be read minus one
Modulus of elasticity temperature factors
Ultimate strengths at temperature intervals (PSI)
Yield point strengths at temperature intervals (PSI)
Coefficients of thermal expansion at temperature intervals (in/in/°F)
Ultimate strains at temperature intervals (in/in)

APPENDIX C

THE PROGRAM

Presented on the following pages is a program, for use with the Burroughs 5500 Computer, that will by numerical methods calculate the temperature gradient and thermal stresses in a free flat plate as it is either cooled or heated from some uniform initial temperature.

```
BEGIN
FILE IN          CHAS(2,10);
FILE OUT         RICH 16 (2,15);
FORMAT OUT       FMT1("TIME",X2,F10.5,X2,"SEC"/),
                 FMT2("LOC",X2,F10.5,X2,"CM",X5,"TEMP",F10.5,X2,"C",X5,
                      "LOC",X2,F10.5,X2,"IN",X5,"TEMP",F10.5,X2,"F"/),
                 FMT3("LOC",X2,F10.5,X2,"IN",X5,"CRACKED"/),
                 FMT4("LOC",X1,F7.3,X1,"IN",X4,"YP",X1,F13.3,X1,"PSI",X4,
                      "STRAIN",X1,F10.8,X1,"IN/IN",X4,"SP",X1,F10.8,X1,
                      "IN/IN",X4,"STRESS",X1,F13.3,X1,"PSI"/);
REAL             A,DX,D,H,T00,TI,KI,CI,DI,DT,HCA,TDT,TDX,T002,H2,AI,S,LH,
                 DDX,TTDX,AAS,EFLP,ELP,AALA,AALB,BB3,BB4,ADI,BB5,BB6,
                 ME,WZ,EQ1,EQ2,EQ3,DEN,AA1,AA2,AA3,BB1,BB2,ALA,ALB,DSUM,
                 KB,KB2,KA,KA2,TA,TA2;
INTEGER          B,G,M,L,N,TN,C,W,TH,Z,E,P,RUN,NP,
                 FAC,CA,J,COUNT;
REAL ARRAY       HCB[0:75],TT[0:75],TB[0:75],TIM[0:50],TEM1[0:50],T[0:75],
                 TEM2[0:50],TEM[0:75],TRANS[0:75],R[0:75],HC[0:75],
                 CTEM[0:75,0:75],CHC[0:75,0:75],TTT[0:75],K[0:50],
                 TTRANS[0:75],AATEM[0:75,0:75],AACTE[0:75,0:75],
                 MEF[0:50],YP[0:50],US[0:50],CTE[0:50],EP2[0:50],
                 AME[0:75],OSU[0:75],AYP[0:75],AUS[0:75],ACTE[0:75],
                 AEP1[0:75],AEP2[0:75],FF[0:75],EXPT[0:75],
                 STRESS[0:75],ERR[0:75],BEXPT[0:75],ADSU[0:75];
INTEGER ARRAY    ACT[0:50];
LABEL            LAB1,LAB2,LAB3,LAB4,LAB5,LAB6,LAB8,LAB16,LAB17,
                 LAB18,LAB21;
                 WRITE(RICH[NO]);
                 READ (CHAS,/,A,DX,T00,T002,TI,D,KI,CI,B,G,W,H,H2,AI,
                 S,E,C);
                 READ (CHAS,/,FOR N ← 0 STEP 1 UNTIL B DO K[N]);
                 READ (CHAS,/,FOR N ← 0 STEP 1 UNTIL E DO TIM[N]);
                 READ (CHAS,/,FOR N ← 0 STEP 1 UNTIL E DO TEM1[N]);
                 READ (CHAS,/,FOR N ← 0 STEP 1 UNTIL E DO TEM2[N]);
                 READ (CHAS,/,FOR N ← 0 STEP 1 UNTIL C DO HC[N]);
                 READ (CHAS,/,ME,AAS,NP);
                 READ (CHAS,/,FOR N ← 0 STEP 1 UNTIL NP DO MEF[N]);
```

```
        READ (CHAS,/,FOR N ← 0 STEP 1 UNTIL NP DO  US[N]) ;
        READ (CHAS,/,FOR N ← 0 STEP 1 UNTIL NP DO  YP[N]) ;
        READ (CHAS,/,FOR N ← 0 STEP 1 UNTIL NP+9 DO CTE[N]) ;
        READ (CHAS,/,FOR N ← 0 STEP 1 UNTIL NP DO EP2[N]) ;
        A ← A×6.4516 ;
        DDX ← DX ;
        DX ← DDX×2.54 ;
        TOO ← ((TOO-32)×(5/9)) ;
        TOO2 ← (TOO2-32)×(5/9) ;
        TI ← (TI-32)×(5/9) ;
        D ← D×27.68 ;
        H ← H×0.000135625712 ;
        H2 ← H2×0.000135625712 ;
        KI ← KI×0.00015 ;
        FOR J ← 1 STEP 1 UNTIL G DO
BEGIN
        DSU[J] ← 0.0 ;
        ADSU[J] ← 0.0 ;
END;
        FOR N ← 0 STEP 1 UNTIL E DO
BEGIN
        TEM1[N] ← ((TEM1[N]-32)×(5/9)) ;
        TEM2[N] ← ((TEM2[N]-32)×(5/9)) ;
END;
        FOR N ← 0 STEP 1 UNTIL C DO TEM[N] ← 50×N ;
        FOR N ← 0 STEP 1 UNTIL B DO
        K[N]←(K[N]×A)/DX ;
        DT ← (CI×D×DX×DX)/(2×KI) ;
        FOR N ← 0 STEP 1 UNTIL G DO TRANS[N] ← 0 ;
        FOR N ← 0 STEP 1 UNTIL G DO ACT[N] ← 0 ;
        IF TI ≤ 722.0 THEN GO TO LAB3 ;
        T[0] ← TOO ;
        FOR N ← 1 STEP 1 UNTIL G DO
        T[N] ← TI ;
COMMENT THE FOLLOWING STATEMENTS UNTIL LABEL LAB3 CALCULATE THE
        PHASE TRANSFORMATION TEMPERATURES ;
        FOR M ← 1 STEP 1 UNTIL W DO
```

```
          BEGIN
               TDT ← M×DT ;
               FOR N ← 1 STEP 1 UNTIL G DO
          BEGIN
               IF N = 1 THEN
          BEGIN
INTEGER         DUMMY1 ;
                KB ← H×A ;
                TA ← (T[N]+T[N+1])/2 ;
                TN ← ENTIER(TA/100) ;
                KA ← ((TA-(TN×100))/100)×(K[TN+1]-K[TN]) ;
                KB2 ← K[TN]+KA ;
                DI ← DT×((KB×(T[0]-T[N]))+(KB2×(T[N+1]-T[N]))) ;
                ADI ← DI/(0.5×DX×D×A) ;
                GO TO LAB1 ;
          END;
               IF N = G THEN
          BEGIN
INTEGER         DUMMY2 ;
                T[G+1] ← TOD2 ;
                TA ← (T[N]+T[N-1])/2 ;
                TN ← ENTIER(TA/100) ;
                KA ← ((TA-(TN×100))/100)×(K[TN+1]-K[TN]) ;
                KB ← K[TN]+KA ;
                KB2 ← H2×A ;
                DI ← DT×((KB2×(T[G+1]-T[N]))+(KB×(T[N-1]-T[N]))) ;
                ADI ← DI/(0.5×DX×D×A) ;
                GO TO LAB1;
          END;
                TA ← (T[N]+T[N-1])/2 ;
                TN ← ENTIER(TA/100) ;
                KA ← ((TA-(TN×100))/100)×(K[TN+1]-K[TN]) ;
                KB ← K[TN]+KA ;
                TA2 ← (T[N]+T[N+1])/2 ;
                TN ← ENTIER(TA2/100) ;
                KA2 ← ((TA2-(TN×100))/100)×(K[TN+1]-K[TN]) ;
                KB2 ← K[TN]+KA2 ;
```

```
                DI ← DT×((KB×(T[N-1]-T[N]))+(KB2×(T[N+1]-T[N]))) ;
                ADI ← DI/(DX×D×A) ;
LAB1:           HCA ← AI ← (S×T[N]) ;
                HCB[N] ← HCA+ADI ;
                R[N] ← (HCB[N]-AI)/S ;
                IF TRANS[N] ≠ 0 THEN GO TO LAB2 ;
                FOR P ← 0 STEP 2 UNTIL (E-1) DO
           BEGIN
                IF (TIM[P]<TDT AND TIM[P+1]≥TDT) AND ((TEM1[P]<R[N] AND
                TEM1[P+1]>R[N]) OR (TEM2[P]<R[N] AND TEM2[P+1]>R[N]))
                THEN
           BEGIN
                TRANS[N] ← R[N] ;
                GO TO LAB2 ;
           END;
           END;
LAB2:      END;
                FOR N ← 1 STEP 1 UNTIL G DO T[N] ← R[N] ;
           END;
                FOR P ← 1 STEP 1 UNTIL G DO
           BEGIN
                IF ( TRANS[P] = 0.0 AND T[P] ≤ 500.0 ) THEN
                TRANS[P] ← 223.0 ;
           END;
LAB3:           T[0] ← TDO ;
                FOR N ← 1 STEP 1 UNTIL G DO
                T[N] ← TI ;
COMMENT         THE FOLLOWING STATEMENTS UNTIL LABEL LAB8 CALCULATE THE
                TEMPERATURE DISTRIBUTION ACROSS THE PLATE ;
                FOR M ← 1 STEP 1 UNTIL W DO
           BEGIN
                TDT ← M×DT ;
                WRITE (RICH,FMT1,TDT);
                FOR N ← 1 STEP 1 UNTIL G DO
           BEGIN
                IF N = 1 THEN
           BEGIN
```

```
INTEGER         DUMMY3 ;
                KB ← H×A ;
                TA ← (T[N]+T[N+1])/2 ;
                TN ← ENTIER(TA/100) ;
                KA ← ((TA-(TN×100))/100)×(K[TN+1]-K[TN]) ;
                KB2 ← K[TN]+KA ;
                DI ← DT×((KB×(T[0]-T[N]))+(KB2×(T[N+1]-T[N]))) ;
                ADI ← DI/(0.5×DX×D×A) ;
                GO TO LAB4 ;
          END;
                IF N = G THEN
          BEGIN
INTEGER         DUMMY4 ;
                T[G+1] ← T002 ;
                TA ← (T[N]+T[N-1])/2 ;
                TN ← ENTIER(TA/100) ;
                KA ← ((TA-(TN×100))/100)×(K[TN+1]-K[TN]) ;
                KB ← K[TN]+KA ;
                KB2 ← H2×A ;
                DI ← DT×((KB2×(T[G+1]-T[N]))+(KB×(T[N-1]-T[N]))) ;
                ADI ← DI/(0.5×DX×D×A) ;
                GO TO LAB4 ;
          END;
                TA ← (T[N]+T[N-1])/2 ;
                TN ← ENTIER(TA/100) ;
                KA ← ((TA-(TN×100))/100)×(K[TN+1]-K[TN]) ;
                KB ← K[TN]+KA ;
                TA2 ← (T[N]+T[N+1])/2 ;
                TN ← ENTIER(TA2/100) ;
                KA2 ← ((TA2-(TN×100))/100)×(K[TN+1]-K[TN]) ;
                KB2 ← K[TN]+KA2 ;
                DI ← DT×((KB×(T[N-1]-T[N]))+(KB2×(T[N+1]-T[N]))) ;
                ADI ← DI/(DX×D×A) ;
LAB4;           IF ACT[N] = 1 THEN GO TO LAB5 ;
                FOR P ← 0 STEP 1 UNTIL G DO
          BEGIN
                IF TRANS[P] = 0 THEN TRANS[P] ← 722 ;
```

```
END;
      FOR P ← 0 STEP 1 UNTIL G DO
      TTRANS[P] ← (TRANS[P]×1.8)+32 ;
      FOR J ← 1 STEP 1 UNTIL G DO
BEGIN
      FOR P ← 0 STEP 1 UNTIL C DO
BEGIN
      CTEM[J,P] ← TEM[P]-(722-TRANS[J]) ;
      CHC[J,P] ← HC[P]-(S×(722-TRANS[J])) ;
      ACT[J] ← 1 ;
END;
END;
LAB5: FOR P ← 0 STEP 1 UNTIL C DO
BEGIN
      IF (CTEM[N,P]<T[N] AND CTEM[N,P+1]≥T[N]) THEN
BEGIN
      HCA ← CHC[N,P]+((CHC[N,P+1]-CHC[N,P])×((T[N]-CTEM[N,P])/
      (CTEM[N,P+1]-CTEM[N,P]))) ;
      HCB[N] ← HCA+ADI ;
      GO TO LAB6 ;
END;
END;
LAB6: FOR P ← 0 STEP 1 UNTIL C DO
BEGIN
      IF (CHC[N,P]<HCB[N] AND CHC[N,P+1]≥HCB[N]) THEN
BEGIN
      TT[N]   ← CTEM[N,P]+((CTEM[N,P+1]-CTEM[N,P])×((HCB[N]-
      CHC[N,P])/(CHC[N,P+1]-CHC[N,P]))) ;
      TDX ← (N-1)×DX ;
      TTDX ← (N-1)×DDX ;
      TTT[N]   ← (TT[N]  ×1.8)+32 ;
      EFLP ← ENTIER(M/5) ;
      ELP ← M/5 ;
      IF EFLP = ELP THEN
      WRITE (RICH,FMT2,TDX,TT[N]  ,TTDX,TTT[N]  ) ;
      GO TO LAB8 ;
END;
```

```
          END;
LAB8:          R[N] ← TT[N]    ;
          END;
               FOR N ← 1 STEP 1 UNTIL G DO T[N] ← R[N] ;
               IF EFLP ≠ ELP THEN GO TO LAB18 ;
COMMENT        THE FOLLOWING STATEMENTS UNTIL LABEL LAB18 CALCULATE THE
               STRESS AND STRAIN DISTRIBUTION ACROSS THE PLATE ;
               FOR N ← 1 STEP 1 UNTIL G DO
          BEGIN
               Z ←ENTIER(TTT[N]  /100) ;
               WZ ← (TTT[N]  -(100×Z))/100 ;
               AME[N] ← (MEF[Z]+((MEF[Z+1]-MEF[Z])×WZ))×ME ;
               AYP[N] ← YP[Z]+((YP[Z+1]-YP[Z])×WZ) ;
               FOR P ← 0 STEP 1 UNTIL NP+9 DO
          BEGIN
               AATEM[N,P] ← (P×100)-(1331.6-TTRANS[N]) ;
               AACTE[N,P] ← CTE[P]-(AAS×(1331.6-TTRANS[N])) ;
          END;
               FOR P ← 0 STEP 1 UNTIL NP+8 DO
          BEGIN
               IF (AATEM[N,P]<TTT[N] AND AATEM[N,P+1]≥TTT[N]) THEN
          BEGIN
               ACTE[N] ← AACTE[N,P]+((AACTE[N,P+1]-AACTE[N,P])×
                    ((TTT[N] - AATEM[N,P])/100)) ;
               GO TO LAB21 ;
          END;
          END;
LAB21:         AUS[N] ← US[Z]+((US[Z+1]-US[Z])×WZ) ;
               AEP1[N] ← AYP[N]/AME[N] ;
               AEP2[N] ← EP2[Z]+((EP2[Z+1]-EP2[Z])×WZ) ;
               FF[N] ← ((AUS[N]-AYP[N])/(AEP2[N]-AEP1[N])) ;
               AYP[N] ← (AEP1[N]+((FF[N]/(AME[N]-FF[N]))×ADSU[N]))×
                    AME[N] ;
               AEP1[N] ← AYP[N]/AME[N] ;
               AEP2[N] ← AEP2[N]-DSU[N] ;
               AUS[N] ← AYP[N]+((AEP2[N]-AEP1[N])×FF[N]) ;
          END;
```

```
      EQ1 ← 0 ;
      EQ2 ← 0 ;
      EQ3 ← 0 ;
      BB1 ← 0 ;
      BB2 ← 0 ;
      BB3 ← 0 ;
      BB4 ← 0 ;
      FAC ← (G-1)/2 ;
      CA ← FAC+1 ;
      FOR J ← -FAC STEP 2 UNTIL (FAC-2) DO
BEGIN
      EQ1 ← EQ1+((DDX/3)×(((AME[J+CA])×((J×DDX)*2))+(4×((
            AME[J+CA+1])×(((J+1)×DDX)*2)))+((AME[J+CA+2])×(
            (( J+2)×DDX)*2)))) ;
      EQ2 ← EQ2+((DDX/3)×(((AME[J+CA])×J×DDX)+(4×(AME[J+CA+1]×
            (J+1)×DDX))+(AME[J+CA+2]×(J+2)×DDX))) ;
      EQ3 ← EQ3+((DDX/3)×(AME[J+CA]+(4×AME[J+CA+1])+
            AME[J+CA+2])) ;
END;
      DEN ← (EQ3×EQ1)-(EQ2*2) ;
      AA1 ← EQ1/DEN ;
      AA2 ← EQ2/DEN ;
      AA3 ← EQ3/DEN ;
      FOR J ← -FAC STEP 2 UNTIL (FAC-2) DO
BEGIN
      BB1 ← BB1+((DDX/3)×((AME[J+CA]×ACTE[J+CA]× TTT[J+CA]×J×
            DDX)+(4×(AME[J+CA+1]×ACTE[J+CA+1]× TTT[J+CA+1]×
            (J+1)×DDX))+(AME[J+CA+2]×ACTE[J+CA+2]× TTT[J+CA+2]
            ×(J+2)×DDX))) ;
      BB2 ← BB2+((DDX/3)×((AME[J+CA]×ACTE[J+CA]× TTT[J+CA])
            +(4×(AME[J+CA+1]×ACTE[J+CA+1]× TTT[J+CA+1]))
            +(AME[J+CA+2]×ACTE[J+CA+2]× TTT[J+CA+2]))) ;
      BB3 ← BB3+((DDX/3)×((AME[J+CA]×DSU[J+CA]×J×DDX)+(4×(
            AME[J+CA+1]×DSU[J+CA+1]×(J+1)×DDX))+(AME[J+CA+2]×
            DSU[J+CA+2]×(J+2)×DDX))) ;
      BB4 ← BB4+((DDX/3)×((AME[J+CA]×DSU[J+CA])+(4×(AME[J+CA+1]
            ×DSU[J+CA+1]))+(AME[J+CA+2]×DSU[J+CA+2]))) ;
```

```
          END;
               ALA  ← (AA1×BB2)+(AA1×BB4)-(AA2×BB1)-(AA2×BB3) ;
               ALB  ← (AA3×BB1)+(AA3×BB3)-(AA2×BB2)-(AA2×BB4) ;
               FOR N ← -FAC STEP 1 UNTIL FAC DO
               EXPT[N+CA] ← ALA+(ALB×N×DDX)-(ACT[N+CA]×  TTT[N+CA])
                    -DSU[N+CA] ;
               FOR J ← 1 STEP 1 UNTIL G DO ERR[J] ← 0.0 ;
LAB17;         FOR N ← 1 STEP 1 UNTIL G DO
          BEGIN
               IF ABS(EXPT[N]) ≤ AEP1[N] THEN
          BEGIN
               STRESS[N] ← AME[N]×EXPT[N] ;
               ERR[N] ← 0.0 ;
               GO TO LAB16 ;
          END;
               STRESS[N] ← ((FF[N]×(ABS(EXPT[N])-AEP1[N]))+AYP[N])
                    ×SIGN(EXPT[N]) ;
               ERR[N] ← (ABS(EXPT[N])-(ABS(STRESS[N])/AME[N]))
                    ×SIGN(EXPT[N]) ;
LAB16;    END;
               BB5 ← 0 ;
               BB6 ← 0 ;
               FOR J ← -FAC STEP 2 UNTIL (FAC-2) DO
          BEGIN
               BB5 ← BB5+((DDX/3)×((AME[J+CA]×ERR[J+CA]×J×DDX)+(4×(
                    AME[J+CA+1]×ERR[J+CA+1]×(J+1)×DDX))+(AME[J+CA+2]×
                    ERR[J+CA+2]×(J+2)×DDX))) ;
               BB6 ← BB6+((DDX/3)×((AME[J+CA]×ERR[J+CA])+(4×(AME[J+CA+1]
                    ×ERR[J+CA+1]))+(AME[J+CA+2]×ERR[J+CA+2]))) ;
          END;
               AALA ← (AA1×BB2)+(AA1×(BB4+BB6))-(AA2×BB1)-(AA2×
                    (BB3+BB5)) ;
               AALB ← (AA3×BB1)+(AA3×(BB3+BB5))-(AA2×BB2)-(AA2×
                    (BB4+BB6)) ;
               FOR J ← -FAC STEP 1 UNTIL FAC DO
          BEGIN
               BEXPT[J+CA] ← EXPT[J+CA] ;
```

```
            EXPT[J+CA] ← AALA+(AALB×J×DDX)-(ACTE[J+CA]×TTT[J+CA])
                  -DSU[J+CA] ;
      END;
            COUNT ← 0 ;
            FOR J ← 1 STEP 1 UNTIL G DO
      BEGIN
            IF ABS(BEXPT[J]-EXPT[J]) > 0.000001 THEN COUNT ← 1 ;
            IF COUNT = 1 THEN GO TO LAB17 ;
      END;
            FOR J ← 1 STEP 1 UNTIL G DO
      BEGIN
            TTDX ← (J-1)×DDX ;
            DSU[J] ← DSU[J]+ERR[J] ;
            ADSU[J] ← ADSU[J]+ABS(ERR[J]) ;
            WRITE (RICH,FMT4,TTDX,AYP[J],BEXPT[J],ERR[J],STRESS[J]) ;
            IF ABS(EXPT[J]) > AEP2[J] THEN
            WRITE (RICH,FMT3,TTDX) ;
      END;
LAB18:      END;
            END.
```

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